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Practical Mathematics

FOR THE

Engineer and Electrician

BY

ELMER E. BURNS

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AND

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PREFACE.

This book is not a course in mathematics, but a handy book for the practical engineer and electrician.

As the book is chiefly intended for the operating engineer and electrician, it treats only of such subjects as the authors have found from their own experience will be of value to such men.

A great many of the best operating engineers and electricians, while knowing their trade well, have never had the advantage of a college education, and the ordinary books on mathematics are written above their heads as well as being too voluminous.

To those who have had a mathematical training, this book will also be of especial value on account of the practical way in which all subjects are treated.

Most books published on technical subjects contain many useless pages, either of theory or of subjects which are of no especial value to the general reader.

It has been the aim of the authors of this book to make it of value to all such men by making it elementary, written in simple, clear language, and treating only of such subjects as are of especial value to all practical workmen.

The authors have assumed that the reader knows but little of mathematics; and, while the subjects selected cover a wide scope, being such as are usually found scattered through two or three different volumes, no subject has been treated which requires knowledge of any preceding subject or any previous knowledge of mathematics.

Any one who fully understands all the subjects here treated will find that, however elementary this book may be considered, he will have all the mathematics that is required to make him a first-class operating engineer or electrical worker.

THE AUTHORS.

Chicago, May, 1912.

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PRACTICAL MATHEMATICS

FOR THE

ENGINEER AND ELECTRICIAN

The engineer and the electrician must be prepared to compute accurately the cost of supplies and the cost of power, to furnish estimates of cost of proposed installations including material and labor, and to work out the problems that arise in every power plant. In this series of articles on practical mathematics we shall give all the methods needed for making such estimates and calculations. We shall begin with

Common Fractions.

A fraction is a part or a certain number of parts, of a unit, as ½ cent, ¾ foot, ⅞ inch, etc.

Take the fraction ⅞ inch. An inch is divided into eight parts. The number below the line tells how many parts the unit is divided into. This number is called the **denominator.** It denominates or names the parts. Each part is one-eighth of an inch. The number above the line tells how many parts are taken. It is called the **numerator,** for it numbers or numerates the parts. If we say that a screw is ⅞ inch long, we suppose an inch to be divided into 8 parts and the screw is as long as 7 of these parts. The name of each part is one-eighth, and the number of parts is 7. The denominator is 8. The numerator is 7.

Multiplying Fractions.

To multiply a fraction by a whole number, we multiply the **numerator** of the fraction by the whole number. For example: if we have three pieces of piping each ¾ of a foot in length, then when the three

pieces are placed end to end, the three together will be 3 times ¾ or ¾ feet in length. Now, since there are 4 fourths in one foot, and 8 fourths is two feet, then ¾ will be 1 fourth more than 2 feet, that is 2¼ feet.

Again, if we buy 50 feet of pipe that costs ¾ cent a foot, the 50 feet will cost 50 times ¾, that is $^{150}/_{4}$ cents. Now, since in 1 cent there are 4 fourths, we find how many times 150 fourths contains 4 fourths. 4 goes into 150, 37 times and 2 remaining, that is there are 37 and 2 fourths cents. Since 4 fourths make a cent, 2 fourths make one-half cent. We have 37½ cents.

$$^{150}/_{4} = 37\frac{1}{2}.$$

Examples:

1. Find the cost of 70 feet of pipe, at ¾ cents a foot.— Answer, 52½ cents.
2. Find the cost of 120 feet of pipe, at ⅞ cents a foot.— Answer, 105 cents, or $1.05.

If we have a number that consists of a whole number and a fraction, to multiply the number we multiply the whole number and the fraction **separately,** and add the results.

Thus to multiply 25¼ by 3 — 3 times 25 equals 75, and 3 times ¼ equals ¾. Adding we have 75 and 3 fourths. Hence, $25\frac{1}{4} \times 3 = 75\frac{3}{4}$, or

$$\begin{array}{r} 25\frac{1}{4} \\ \times\quad 3 \\ \hline 75\frac{3}{4} \end{array}$$

To multiply 27½ by 3, 3 times 27 equals 81, and 3 times ½ equals ⅔. But ⅔ equals 1½. Adding 81 and 1½ we have 82½. Hence, $27\frac{1}{2} \times 3 = 82\frac{1}{2}$, or written in another way,

$$\begin{array}{r} 27\frac{1}{2} \\ \times\quad 3 \\ \hline 82\frac{1}{2} \end{array}$$

In the last multiplication we first multiply ½ by 3. Three times ½ equals 1½. We put down ½ and carry the 1 to add to the next figure. 3 times 7 are 21 and 1 to add makes 22. Put down 2 in units place and carry the 2 in tens place or twenty. 3 times 2 are 6 and 2 to add makes 8. Write 8 in tens place, then we have the answer, 82½.

Examples:

3. In a certain heating plant an average of 15½ tons of coal per day is used. How many tons are used in 6 days? — Answer, 93 tons.

4. What will be the cost of this coal at $4 per ton? — Answer, $372.

5. A certain wire with its insulation is 3/16 inch in thickness. If this wire is wound on a spool until there are 5 layers, what will be the thickness of the 5 layers? — Answer, 15/16 inch.

6. What will be the cost of 40 pounds of annunciator wire at 30½ cents a pound? —Answer, 1,220 cents, or $12.20.

7. What will be the total length of 4 pieces of pipe connected end to end if each piece is 3½ feet in length? —Answer, 14 feet.

8. If coal cost $4½ per ton, what will be the cost of 75 tons? —Answer, $337½.

9. What will be the cost of 75 feet of No. 14 cable at 12½ cents a foot? —Answer, $9.37½.

10. 250 feet of wire, at 12½ cents a foot; 8 switches, at 18½ cents each, and 6 sockets, at 12½ cents each, are used in a wiring job. What is the total cost? —Answer,

wire,	$31.25
Switches,	1.48
Sockets,	.75
Total,	$33.48

Dividing Fractions.

We **multiply** a fraction by multiplying the **numerator.**

We **divide** a fraction by dividing the **numerator.**

Whatever you wish to do to a fraction, do it to the numerator. Whatever is done to the numerator is done to the fraction. This simple rule will guide you in all calculations with fractions.

For example: Divide $\frac{8}{15}$ by 2.

8 divided by 2 equals 4.

$\frac{8}{15}$ divided by 2 equals $\frac{4}{15}$.

$\frac{8}{15} \div 2 = \frac{4}{15}$.

Sometimes we can not divide the numerator without changing the fraction.

For example: Divide $\frac{7}{8}$ by 2.

Seven can not be divided by 2 without a remainder. We want a number for the numerator that can be divided without a remainder. We can change the fraction by multiplying both numerator and denominator by the same number. This will not change the value of the fraction. Multiplying both terms of the fraction $\frac{7}{8}$ by 2 we get $\frac{14}{16}$. We have not multiplied the fraction. We have not changed its value. $\frac{14}{16}$ is the same as $\frac{7}{8}$. $\frac{14}{16}$ of an inch is exactly the same length as $\frac{7}{8}$ of an inch. (Fig. 1.)

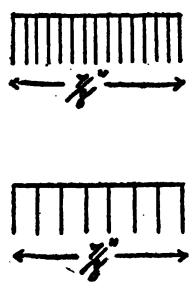


FIG. 1.

Now we can divide the fraction $^{14}/_{16}$ by 2, according to the rule given above.

$$^{14}/_{16} \div 2 = {}^{7}/_{16}.$$

We started out to divide $^{7}/_{8}$ by 2 and we reach the result $^{7}/_{16}$. Hence, $^{7}/_{8} \div 2 = {}^{7}/_{16}$.

It is plain that if we multiply the denominator of the fraction $^{7}/_{8}$ by 2 we get $^{7}/_{16}$. Therefore, if we wish to divide a fraction and can not divide the numerator we **multiply** the denominator.

Examples:

1. Divide $^{15}/_{16}$ by 3.—Answer, $^{5}/_{16}$.
2. Divide $^{15}/_{16}$ by 5.—Answer, $^{3}/_{16}$.
3. Divide $^{8}/_{9}$ by 2.—Answer, $^{4}/_{9}$.
4. Divide $^{3}/_{5}$ by 2.—Answer, $^{3}/_{10}$.
5. Divide $^{7}/_{16}$ by 3.—Answer, $^{7}/_{48}$.

To divide a **mixed number**, that is, a number which contains a whole number and a fraction, we divide the whole number and the fraction separately if the whole number can be divided without a remainder.

For example, divide $21\frac{3}{4}$ inches by 3. 21 divided by 3 equals 7. $\frac{3}{4}$ divided by 3 equals $\frac{1}{4}$. The result is $7\frac{1}{4}$ inches.

$$21\frac{3}{4} \div 3 = 7\frac{1}{4}.$$

Examples:

6. Divide $10\frac{5}{6}$ by 2.—Answer, $5\frac{1}{6}$.
7. Divide $15\frac{5}{8}$ by 5.—Answer, $3\frac{1}{8}$.
8. Divide $18\frac{3}{8}$ by 3.—Answer, $6\frac{1}{8}$.

If the whole number can not be divided without a remainder the simplest way usually is to change the entire quantity to a fraction and then divide.

For example, divide $3\frac{1}{5}$ inches by 2. In one inch there are 5 fifths. In three inches there are 3 times 5 fifths, or 15 fifths.

$$1 = {}^{5}/_{5}$$
$$3 = {}^{15}/_{5}$$

Therefore, $3\frac{1}{5} = {}^{15}/_{5} + {}^{1}/_{5} = {}^{16}/_{5}$.

This is clear if we have 3 inches, each inch divided into fifths and then 4 fifths of an inch more. (Fig. 2.) In the three inches there are 15 fifths, and 4 fifths more make, in all, 19 fifths.

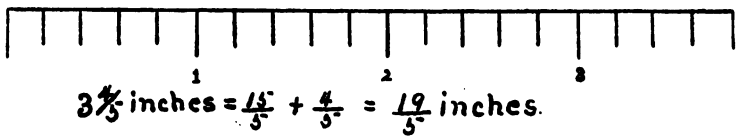


FIG. 2.

Now we can divide by 2 by the rule given above.

$$^{19}\!/\!_{5} \div 2 = {}^{19}\!/\!_{10}.$$

But there are 10 tenths in one inch. Hence, $^{19}\!/\!_{10}$ make one inch and $^{9}\!/\!_{10}$ over.

$$^{19}\!/\!_{10} = 1{}^{9}\!/\!_{10}.$$

Shortening the work we have:

$$3{}^{4}\!/\!_{5} \div 2 = {}^{19}\!/\!_{5} \div 2 = {}^{19}\!/\!_{10} = 1{}^{9}\!/\!_{10},$$

or, $3{}^{4}\!/\!_{5} \div 2 = 1{}^{9}\!/\!_{10}$.

Examples:

9. Divide $5{}^{4}\!/\!_{5}$ by 3.—Answer, $^{29}\!/\!_{15} = 1{}^{14}\!/\!_{15}$.
10. Divide $7{}^{3}\!/\!_{8}$ by 2.—Answer, $^{59}\!/\!_{16} = 3{}^{11}\!/\!_{16}$.
11. Divide $10{}^{3}\!/\!_{7}$ by 3.—Answer, $^{73}\!/\!_{21} = 3{}^{10}\!/\!_{21}$.

To multiply or divide one fraction by another fraction:

To multiply one fraction by another, multiply the numerators together and write this product as the numerator of the answer. Then multiply the denominators together and write this product as the denominator of the answer.

Thus, if we have ¼ of an inch and wish to multiply it by ½, that is, to find one-half of one-fourth, we have as the answer ⅛ inch.

$$¼ \times ½ = ⅛.$$

If we had a drill that would drill a hole ¼ inch across, one-half the diameter of this hole would be ⅛ inch.

Multiply ⅜ by ½.

$$\frac{3}{8} \times \frac{1}{2} = \frac{15}{16}.$$

Multiply ¾ by ⅔.

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}.$$

We can see that 6/12 equal ½ if we divide an inch into twelfths. A half inch, then, contains 6 twelfths. If we divide both terms of the fraction 6/12 by 6 we get ½. (Fig. 3.)

Dividing both terms of a fraction by the same number does not change the value of the fraction.

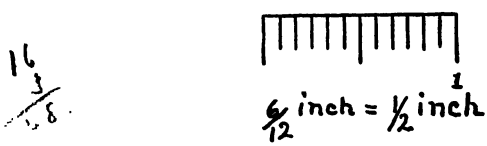


FIG. 3.

Examples:

1. Multiply ⅞ by ⅔.—Answer, 14/24 = 7/12.
2. Multiply ⅝ by 5/7.—Answer, 3/7.
3. Find two-thirds of 5/16 of an inch.—Answer, 10/48 = 5/24.

To divide one fraction by another, invert the divisor and then multiply.

For example:

Divide ⅝ by ⅔.

$$\frac{5}{8} \div \frac{2}{3} = \frac{5}{8} \times \frac{3}{2} = \frac{15}{16}.$$

Divide ⅞ by 1/16.

$$\frac{7}{8} \div \frac{1}{16} = \frac{7}{8} \times \frac{16}{1} = \frac{112}{8} = 4.$$

The last example is clear if we compare ⅔ of an inch and 1/16 of an inch. We can see that 1/16 of an inch is contained 4 times in ⅞ of an inch.

Examples:

4. Divide ½ by ⅛.—Answer, $\frac{8}{2} = 1\frac{1}{2}$
5. Divide ⅛ by ½.—Answer, ⅓.
6. Divide $\frac{12}{15}$ by ¾.—Answer, $\frac{12}{15} = \frac{4}{5}$.
7. Divide ¾ by ⅜.—Answer, $\frac{6}{4} = 1\frac{1}{4}$.

Adding and Subtracting Fractions.

We have seen that a common fraction denotes a certain number of equal parts into which a thing has been divided. Thus, if an inch is divided into 16 equal parts and we take 5 of those parts, we have five-sixteenths (5/16) of an inch. If a gallon is divided into 4 parts and we take 3 of those parts, we have three-fourths (¾) of a gallon. The number below the line, which we call the denominator, tells us the number of parts into which the thing is divided. It also tells us the size of the parts. Thus, in the first example, each part is 1/16 of an inch. The number above the line tells how many parts are taken. It numbers or numerates the parts; so it is called the numerator.

Now in order to add fractions, the parts must be of the same size, that is, the denominators must be the same. For example, if we wish to add 5/16 of an inch and ⅜ of an inch, we must first find how many sixteenths there are in ⅜. One-eighth is equal to 2/16. Then ⅜ equal three times 2/16, or 6/16. So we have the rule: **To change a fraction to one having a larger denominator, multiply the denominator by a number which will give the required denominator, and also multiply the numerator by the same number.** Multiplying both numerator and denominator by the same number does not change the value of the fraction. To reduce a fraction to one having a smaller denominator, divide both numerator and denominator by a number which will give the required denominator. For example: to reduce 9/12 to fourths, divide both numerator and denominator by 3, and we have $\frac{9}{12} = \frac{3}{4}$.

Suppose we wish to add $\frac{9}{12}$ and $\frac{1}{4}$. We have $\frac{9}{12} = \frac{3}{4}$ and $\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$.

Thus, to add fractions having the same denominator, we add the numerators and write the sum over the denominator.

Suppose we wish to subtract $\frac{1}{4}$ from $\frac{5}{16}$. We have $\frac{1}{4} = \frac{4}{16}$, and $\frac{5}{16}$ less $\frac{4}{16} = \frac{1}{16}$.

Thus, to subtract one fraction from another we change the fractions so that they have the same denominator and subtract the numerator of one from the numerator of the other.

As we have said before, whatever we wish to do to the fractions we do it to the numerators. We must remember, however, that the **denominators must be the same** before we can **add** or **subtract.**

Sometimes we must take an entirely new denominator. For example, if we wish to add $\frac{2}{3}$ and $\frac{1}{5}$ we can not reduce thirds to fifths nor fifths to thirds, but we can reduce both to fifteenths.

$$\frac{2}{3} = \frac{10}{15} \text{ and } \frac{1}{5} = \frac{3}{15},$$
$$\text{then } \frac{10}{15} + \frac{3}{15} = \frac{13}{15}.$$

To add whole numbers and fractions, add the fractions and whole numbers separately. Then add the results together. Thus, to add $25\frac{1}{8}$ and $10\frac{3}{8}$, we have 25 added to 10 equals 35, and $\frac{1}{8}$ added to $\frac{3}{8}$ equals $\frac{4}{8}$. The sum is $35\frac{4}{8}$. We write it thus:

$$\begin{array}{r} 25\frac{1}{8} \\ 10\frac{3}{8} \\ \hline 35\frac{4}{8} \end{array}$$

If the fractions added together make more than one unit, then we must add the number of units made by the fractions to the whole numbers. For example:

Add 22⅝ and 11⅞.

$$\begin{array}{r} 22\frac{5}{8} \\ 11\frac{7}{8} \\ \hline 33\frac{12}{8} \end{array}$$

Result $33\frac{12}{8}$

But $\frac{12}{8} = 1\frac{4}{8}$, so we have

$$\begin{array}{r} 33 \\ 1\frac{4}{8} \\ \hline 34\frac{4}{8} \end{array}$$

This should be done in one operation. Thus:

$$\begin{array}{r} 22\frac{5}{8} \\ 11\frac{7}{8} \\ \hline 34\frac{4}{8} \end{array}$$

We can carry the rest of the process in our heads. We think ⅝ added to ⅞ gives $\frac{12}{8}$. $\frac{12}{8}$ equals $1\frac{4}{8}$. Then we write down the $\frac{4}{8}$ and add the 1 to the 11 and 22.

Rule for Adding or Subtracting Fractions.

The fractions must have the same denominator. Change them if necessary, so that the denominators are the same. Then add or subtract the numerators and write the result over the denominator.

Examples:

1. Add ⅝ and $\frac{1}{8}$.—Answer, $\frac{6}{8} = \frac{3}{4}$.

2. If one can contains ⅔ gallon of oil, and another contains ¾ gallon, how many gallons of oil are there in the two cans?—Answer, $\frac{17}{12} = 1\frac{5}{12}$.

3. If a piece of conduit 6½ feet in length is needed and you have a piece 8⅞ feet in length, how much must be cut off?—Answer, $2\frac{3}{8}$ feet.

4. Subtract $5\frac{2}{3}$ from $6\frac{1}{4}$.

$\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$.

We can not subtract $\frac{8}{12}$ from $\frac{3}{12}$, so we say $6 = 5 + 1\frac{12}{12}$. Then we can subtract $\frac{8}{12}$ from $1\frac{12}{12}$.

$1\frac{12}{12} = \frac{15}{12}$ and

$\frac{15}{12} - \frac{8}{12} = \frac{7}{12}$.

We have taken one away from the 6, so we have left only 5, and five subtracted from five leaves nothing. Hence

$$6\frac{1}{4} - 5\frac{2}{3} = \frac{7}{12}.$$

It is usually written as follows:

$$\begin{array}{ll} 6\frac{1}{4} & 6\frac{3}{12} \\ 5\frac{2}{3} & 5\frac{8}{12} \\ \hline & \hline \\ & \frac{7}{12}. \end{array}$$

We carry the rest of the operation in our heads. We say $\frac{8}{12}$ can not be taken from $\frac{3}{12}$. So we must take 1 from the 6, and add to the $\frac{3}{12}$, making $\frac{15}{12}$. Then $\frac{8}{12}$ from $\frac{15}{12}$ equals $\frac{7}{12}$, and 5 from 5 equals 0. So the result is $\frac{7}{12}$.

5. Subtract $10\frac{2}{3}$ from $12\frac{1}{16}$.—Answer, $1\frac{3}{16}$.

6. Two pieces of piping are to be connected together, end to end. Their lengths are 4 feet $10\frac{1}{2}$ inches, and 6 feet $9\frac{1}{4}$ inches. What will be the length of the two together?—Answer, 11 feet and $7\frac{3}{4}$ inches.

7. The length of a cylinder inside is $16\frac{3}{4}$ inches. The length of stroke of the piston is $16\frac{5}{16}$ inches. What is the clearance?—Answer, Clearance at both ends together $\frac{7}{16}$ inch. Clearance at each end $\frac{7}{32}$ inch.

8. The pressure of steam at four points in the stroke is $56\frac{1}{8}$, $54\frac{3}{8}$, $32\frac{5}{8}$ and $11\frac{1}{4}$ pounds. Find the average of the four pressures. First add the four numbers, then divide by four.—Answer, Sum of pressures, $154\frac{3}{8}$ pounds. Average, $38\frac{19}{32}$ pounds.

9. If a piece 1 foot $6\frac{1}{4}$ inches long is cut from a piece of piping 8 feet long, what length is left?—Answer, 6 feet $5\frac{3}{4}$ inches.

10. The outside diameter of a piece of tubing is $1\frac{5}{16}$; the inside diameter is $\frac{15}{16}$ inch. What is the thickness of the wall of the tubing?—Answer, $\frac{1}{2}$ of $\frac{5}{16}$, or $\frac{5}{16}$ inch.

Decimal Fractions.

Decimal fractions are not difficult. Any one who can make change, who can reduce dollars to cents and cents to dollars, can easily learn to work any problem in decimals. The most familiar example of decimals is our money system. A dime is one-tenth of a dollar, and a cent is one-tenth of a dime. A mill is one-tenth of a cent. Mills are used in estimating taxes.

The word **decimal** means one-tenth. In using decimals we count by tens. In a whole number we also count by tens. The first place is units; the next tens; the next, hundreds, and so on. Each place in a whole number has ten times the value of the place to the right. Thus in the number 666, the six on the right means 6 units, the next 6 is worth ten times as much as the first 6. It means 6 times 10 or 60. The third 6 is worth ten times as much as the second. It means 6 times 100, or 600.

So in a decimal we continue to count by tens. The decimal 0.6 means 6 times one-tenth of a unit, or six-tenths. The decimal 0.06 is worth only one-tenth as much as six-tenths. It means six-hundredths. The decimal 0.006 means six-thousandths, or one-tenth as much as six-hundredths. Thus moving a figure one place to the right makes its value one-tenth as great. Moving a figure one place to the left makes its value ten times as great. Take, for example, $600.00. Move the 6 one place to the right, and we have $60.00, or one-tenth as much as at first. Take $0.60; move the 6 one place to the right, and we have $0.06, or one-tenth as much.

Any operation with decimals, multiplying, dividing, adding, or subtracting, is worked in exactly the same way as with whole numbers, only we must remember to keep the decimal point in its proper place.

Multiplying Decimals.

Multiply $0.65 by 3.

$$\begin{array}{r} \$0.65 \\ 3 \\ \hline \$1.95 \end{array}$$

Multiply $0.65 by 30.

$$\begin{array}{r} \$0.65 \\ 30 \\ \hline \$19.50 \end{array}$$

We can see, then, that when we multiply a decimal by a whole number the number of decimal places in the answer must be the same as the number of decimal places in the number to be multiplied.

Multiply 55 by .3.

$$\begin{array}{r} 55 \\ .3 \\ \hline 16.5 \end{array}$$

In multiplying a whole number by a decimal, there must be as many decimal places in the answer as in the multiplier.

Multiply 0.5 by 0.3.

One-tenth of 0.5 is five-hundredths. One-tenth of $0.50, or five-tenths of a dollar, is $0.05, or five-hundredths of a dollar. Three-tenths of $0.50 is 3 times as much as one-tenth of $0.50, so $0.50 $\times$ 0.3 = $0.15.

So we see that in multiplying a decimal by a decimal there must be as many decimal places in the answer as there are decimal places in **both** the multiplier and the number multiplied taken together.

Dividing Decimals.

Decimals are divided exactly as whole numbers are divided. Great mistakes are often made, however, by putting the decimal point in the wrong place. It is necessary to learn the rules for locating the decimal point.

1. If the divisor and the number divided have the same number of decimal figures there are no decimal figures in the answer.

Example:

Divide $6.25 by $1.25.

```
1.25 ) 6.25 ( 5 answer.
       6.25
```

2. If the dividend or number divided has more decimal figures than the divisor, then find how many more decimal figures the dividend has than the divisor and this number will be the number of decimal figures in the answer.

Example:

If there are three decimal figures in the dividend and one in the divisor there will be two decimal figures in the answer.

Divide 30.225 by 1.5.

```
1.5 ) 30.225 ( 20.15 answer.
      30
      ----
       22
       15
      ----
       75
       75
      ----
```

3. If a decimal is divided by a whole number the

answer will have as many decimal figures as the decimal which is divided.

Example:

```
15 ) 30.225 ( 2.015
     30
     ---
      22
      15
      --
       75
       75
       --
```

Examples to be solved:

1. Multiply $5.50 by .3.—Answer, $1.650.
2. Multiply 450 by .6.—Answer, 270.0.
3. Multiply 0.45 by 25.—Answer, 11.25.
4. How much is fifteen-hundredths of $425, that is $425. multiplied by 0.15?—Answer, $63.75.
5. Five per cent of a number is the same as five-hundredths of the number. How much is five per cent of $350?—Answer, $17.50.
6. In the same way find six per cent or six-hundredths of $3,525.—Answer, $211.50.
7. If property is taxed 7 mills on the dollar, what is the tax on property assessed at $2,000?—Answer, $14.
8. If the tax levy is 8½ mills on the dollar, what is the tax on $3,500? Multiply by .0085.—Answer, $29.75.
9. Find five and one-half per cent of $350. Multiply by .055.—Answer, $19.75.
10. Divide 61.5 by 1.5.—Answer, 41.
11. Divide 61.65 by 1.5.—Answer, 4.1. 41.1
12. If coal cost $2.25 a ton and $240.75 is paid for coal, how many tons are bought?—Answer, 107.

13. Divide $302.25 by 5.—Answer, $60.45.
14. Divide $302.25 by 6.—Answer, $50.375.
15. How many tons of coal at $1.75 a ton can I buy for $73.00?—Answer, 41 tons.
16. What is the weight in pounds of 6.8 tons of coal?—Answer, 13,600 pounds.
17. At $2.75 a ton what is the cost of 50 tons of coal?—Answer, $135.00.

Adding and Subtracting Decimals.

Rule: To add or subtract decimals, write units of the same order in same column. Then add or subtract, as with whole numbers. The decimal point in the answer will be directly under the decimal point of the numbers.

Example:

Add $308.25
450.565
250.25

$1009.065 Answer.

Subtract 355.25 from 1500.75.

$1500.75
355.25

$1145.50 Answer.

Examples to be solved.

1. Add $250.55, $340.65, $75.15.—Answer, $666.35.
2. Add $56.25, $105.565, $240.50.—Answer, $402.315.
3. Subtract $85.75 from $125.55.—Answer, $39.80.
4. Subtract $155.655 from $278.70.—Answer, $123.045.
5. Subtract $256.255 from $1075.555.—Answer, $819.30.

Use of Decimal Fractions in Finding Circumference and Area of a Circle.

The engineer should know how to find the horsepower of a steam engine. In order to do this it is necessary to find the number of square inches in the piston; that is, to find the area of the end of the piston, which is a circle. He should also know how to find the contents of a tank or cistern, and to do this he must first find the area of the bottom of the tank, which is usually a circle. In finding the area of a circle, decimals are used. We have learned how to

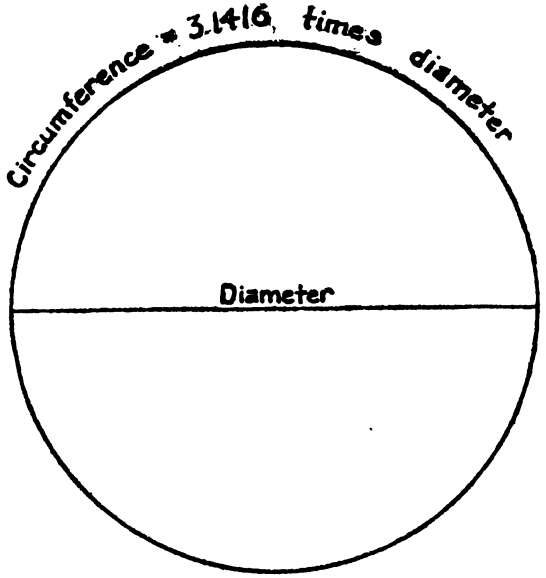


FIG. 4.

multiply decimals; we shall now make use of this knowledge in finding the areas and circumferences of circles.

The diameter is a straight line drawn through the center of a circle. It is the longest straight line that can be drawn in a circle. To find the circumference of a circle multiply the diameter by 3.1416. (Fig. 4.)

Example:

Find the circumference of a circle whose diameter is 11 inches. The example is worked as follows:

$$\begin{array}{r} 3.1416 \\ 11 \\ \hline 31416 \\ 31416 \\ \hline 34.5576 \end{array}$$

There are 34 inches and the decimal .5576 of an inch in the circumference. Remember that we multiply just as though we were multiplying two whole numbers, and then point off as many decimal places in the product as there are decimal places in the multiplier and the number multiplied taken together.

Let us next find the circumference of a piston whose diameter is 16.5 inches.

$$\begin{array}{r} 3.1416 \\ 16.5 \\ \hline 157080 \\ 188496 \\ 31416 \\ \hline 51.83640 \end{array}$$

A string 51.8 inches long would almost reach around the piston. We point off five decimal places in the product because there are four decimal places in

the number multiplied and one decimal place in the multiplier, making five decimal places in all.

The radius of a circle is one-half the diameter.

To find the area of a circle multiply the radius by itself and this product by 3.1416.

Example:

Find the area of a piston whose diameter is 16.5 inches.

Solution: Radius = ½ of 16.5 = 8.25.

$$\begin{array}{r} 8.25 \\ 8.25 \\ \hline 4125 \\ 1650 \\ 6600 \\ \hline 68.0625 \end{array}$$

Suppose we have measured to hundredths of an inch. We must keep three decimal figures. The rule for the number of decimal figures to be retained is always keep one more figure than the smallest figure that is accurately known. If we have measured to hundredths of an inch, we must keep the decimal figure in thousandths place.

So we multiply as follows:

$$\begin{array}{r} 3.1416 \\ 68.062 \\ \hline 62832 \\ 188496 \\ 251328 \\ 188496 \\ \hline 213.8235792 \end{array}$$

The answer is 213.82 square inches. We drop all the figures after hundredths place.

We may also find the area by squaring the diameter and multiplying by .7854.

If the figure in thousandths place had been 5 or more we should have added 1 to the figure in hundredths place and written the answer 213.83.

Examples to be solved:

1. Find the circumference of a piston whose diameter is twelve inches.—Answer, 37.6992 inches.

Dropping unnecessary figures, we have 37.7 inches.

2. Find the area of the same piston.—Answer, 113.1 square inches.

3. Find the circumference of a piston whose diameter is 14.5 inches.—Answer, 45.55 inches.

4. Find the area of the same piston.

Solution:

Radius = 7.25 inches.

$$\begin{array}{r} 7.25 \\ 7.25 \\ \hline 3625 \\ 1450 \\ 5075 \\ \hline 52.5625 \end{array}$$ square of radius.

For practical purposes we may cut off the last two figures in the square of the radius. Multiplying 52.56 by 3.1416 we get 165.12 square inches. The complete answer is 165.122396, but the last figures are of no value since they are less than one-hundredth of a square inch, so we drop them.

5. What is the area in square feet of the bottom of a circular tank if the diameter is 12 feet?—Answer, 113.1 square feet.

6. What is the circumference of the same tank?—Answer, 37.7 feet.

7. The diameter of a piston is 16 inches, the pressure of steam on the piston is 50 pounds on every square inch. What is the total pressure on the piston?

First find the number of square inches in the piston. Then multiply this number by 50, which is the pressure per square inch.—Answer, 10053.12 pounds.

8. The diameter of the piston of an hydraulic elevator is 14 inches. The pressure of the water is 40 pounds on every square inch. What is the total lifting force on the elevator?—Answer, 6157.5 pounds.

A rough and ready method of finding the area and the circumference of a circle is to use the number 3⅐ instead of 3.1416. This method is good enough if we are not measuring to so fine a point as hundredths of an inch.

Example:

Find the circumference and the area of a piston whose diameter is 14 inches.

```
  14
  3⅐
 ----
  42
   2
 ----
  44   inches = circumference

   7
   7
 ----
  49
  3⅐
 ----
 147
   7
 ----
 154   square inches = area.
```

Find the circumference of a circle whose diameter is 15 inches.

$$\begin{array}{r} 15 \\ 3\frac{1}{7} \\ \hline 45 \\ 2\frac{1}{7} \\ \hline 47\frac{1}{7} \end{array}$$ inches = circumference.

In finding the area of this circle it is better to use decimals. Square 7.5 and multiply by 3.1416.

Example to be solved:

9. Find the circumference and area of a circle whose diameter is ten inches.—Answer, Circumference, $31\frac{3}{7}$ inches. Area, $78\frac{4}{7}$ square inches.

Reducing Common Fractions to Decimals and Decimals to Common Fractions.

A decimal is a fraction whose denominator is ten, one hundred, one thousand, or some other power of ten. By **power of ten** we mean a number that you can obtain by multiplying ten by ten and that result by ten, and continue to multiply by ten and never by anything else until you get for a result the number that is given. One hundred is the second power of ten, for you can get 100 by multiplying 10 by 10. Two tens multiplied give 100. One thousand is the third power of 10. One with zeros after it forms a number which is a power of ten.

The denominator of a decimal is known by the position of the decimal point. Thus 0.3 means three-tenths. We can write it $\frac{3}{10}$ and we have a common fraction. 0.15 means fifteen-hundredths. If we write it $\frac{15}{100}$ we have changed the decimal to a common fraction. 0.125 changed to a common fraction is $\frac{125}{1000}$.

Whenever we write the denominator of a decimal instead of using the decimal point, we have a common fraction.

We can reduce the fraction to one having a smaller denominator whenever the numerator and denominator can both be divided by the same number. Thus the numerator and denominator of $^{15}/_{100}$ can both be divided by 5. Dividing both terms of the fraction by 5 we have

$$^{15}/_{100} = {}^{3}/_{20}.$$

In the last example above 125 and 1000 can both be divided by 25. Dividing we get

$$^{125}/_{1000} = {}^{5}/_{40}.$$

Both terms of $^{5}/_{40}$ can be divided by 5.

$$^{5}/_{40} = {}^{1}/_{8}.$$

Therefore, $0.125 = {}^{125}/_{1000} = {}^{1}/_{8}$.

To reduce a common fraction to a decimal we divide the numerator by the denominator.

Example:

Reduce $^{3}/_{5}$ to a decimal.

$^{3}/_{5}$ means 3 divided by 5. We divide in this way.

$$\begin{array}{r} 5\,)\,3.0 \\ \hline .6 \end{array}$$

The dividend is 3 units or 30 tenths, and 30 tenths divided by 5 equals 6 tenths.

To reduce a common fraction to a decimal, annex zeros to the numerator, placing the decimal point before the zeros. Then divide by the denominator, as in division of decimals.

It may happen that even after annexing zeros the numerator can not be exactly divided by the denominator. In this case we carry the division as far as we need to and omit the rest of the decimal.

Example:

⅓ of an inch equals how many hundredths of an inch?

$$3\overline{)1.00}\quad .33+$$

There is a remainder as far as we carry the division, but as we are measuring only to hundredths of an inch we carry the division only to hundredths. The sign + is sometimes placed after such a decimal to show that the division might have been carried farther.

Examples:

1. Reduce 0.7 to a common fraction.—Answer, $\frac{7}{10}$.
2. Reduce 0.65 to a common fraction.—Answer, $\frac{13}{20}$.
3. Reduce $\frac{3}{5}$ to a decimal.—Answer, .4.
4. Five-eighths of a dollar equals how many cents. —Answer, $0.625, or 62½ cents.
5. Reduce 15⅜ to a decimal.—Answer, 15.375.
6. The diameter of No. 0000 wire B. & S. gauge is 0.46 inch. What is the diameter in common fractions? Answer, $\frac{46}{100}$, or $\frac{23}{50}$ inch.
7. The diameter of a pipe is found to be 3⅞ inches. What is the diameter in decimals?—Answer, 3.875 inches.

Percentage.

Percentage is a particular case of decimal fractions. In percentage the denominator is always one hundred. When we find a certain per cent of a number, we find that many hundredths of the number. Thus six per cent means six hundredths. To find six per cent of a number we multiply the number by 0.06.

Examples:

1. Twenty per cent of a number is what fraction of the number?—Answer, $\frac{20}{100}$, or ⅕.

2. One-eighth of a number is what per cent of the number? — Reduce ⅛ to hundredths.—Answer, 12½ per cent.

3. Thirty-three and one-third per cent of a number is what fraction of the number? —Answer ⅓.

4. Find 6 per cent of $1,000.—Answer, $60.00.

5. If an engine wastes in friction 25 per cent of the power it receives, and it receives 80 horse-power, how many horse-power does it waste? —Answer, 20 horse-power.

6. If an electric motor gives out 65 per cent of the electrical power it receives, and receives 12 horse-power, how many horse-power does it give out? — Answer, 7.8 horse-power.

7. If a dynamo gives out in electrical horse-power 90 per cent of the power it receives, and it receives 5,000 horse-power, how many electrical horse-power does it give out? —Answer, 4,500 electrical horse-power.

8. The efficiency of a motor is 80 per cent, that is, it gives out in useful power 80 per cent of the power it receives. It receives 10½ horse-power. What power does it give out? —Answer, 8.4 horse-power.

9. If the power factor of an alternating current is 86.6, that is, the true power is 86.6 per cent of the apparent power, what is the true power if the apparent power is 4,000 kilowatts? Multiply 4,000 by 0.866.— Answer, 3464 kilowatts.

10. What is the true power if the apparent power is 10,500 kilowatts and the power factor 93.97?— Answer, 9866.85 kilowatts. In the last example we multiply 10,500 by 0.9397.

Finding the Horse-power of an Engine.

In finding the horse-power of an engine, the area of the piston must first be found. The end of the piston

is a circle and you have learned how to find the area of a circle. The area must be found in square inches. Measure the diameter of the piston in inches and find the area in square inches.

Next the pressure per square inch must be found. It will not do to take the pressure as shown by the pressure gauge, for this is the pressure of the steam in the boiler. We want the pressure of the steam on the piston, and this is shown by the indicator card. The indicator card shows that the pressure changes, being greatest near the beginning of the stroke and dropping after the cut-off. Now, since there are so many different pressures during a stroke, which pressure shall we take? The answer is we must take the **average** pressure.

To find the **average** pressure, we divide the indicator diagram into a number of parts (say ten), as shown in Fig. 5, and draw vertical lines to the curve. These vertical lines are the pressure lines for the different parts of the stroke. They should be drawn in the middle of the division. They should begin on the line which represents the back pressure, that is, the lower line of the indicator diagram.

Two vertical lines should be drawn just touching the extreme ends of the diagram. The space between these two lines should be divided into ten or twenty equal parts and from the middle point of each of these parts the pressure lines should be drawn as described above. In Fig. 5, A C and B D are the vertical lines just touching the ends of the diagram. The space A B is divided into ten equal parts. From the middle of each part, and perpendicular to the line A B a pressure line is drawn. In the figure the pressure lines are dotted. The part of the pressure line within the curve represents the difference between the forward pressure and the back pressure. This is the pressure that counts in measuring power. We must measure the length of

each pressure line in inches and multiply its length by the scale of the spring.

Fig. 5 is an indicator-card from the crank end of the engine. The average pressure for this card should be found, also the average pressure for a card taken from the head end. The average of these two pressures is the average effective pressure for one revolution.

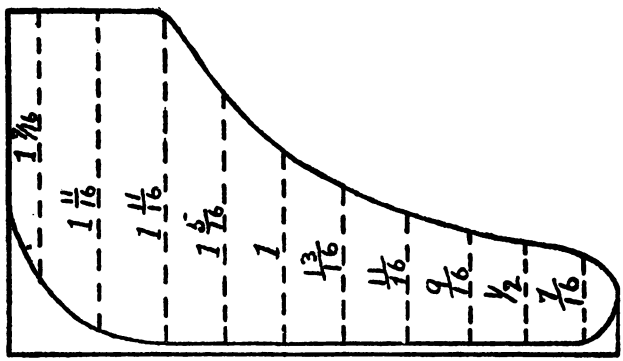


FIG. 5.

For example, if we are using a 40-pound spring, each inch represents 40 pounds of pressure and we multiply by 40. Thus, if the length of a pressure line at a certain point is 1.1 inches, the pressure at that point is 1.1 times 40, which equals 44 pounds. That is to say, 44 pounds is the pressure per square inch when the piston is at that particular point in the stroke. If we find in this way the pressure shown by each pressure line, add all these pressures together and divide by the number of pressure lines, we have the **average** pressure per square inch for the entire stroke. The greater the number of pressure lines used, the more accurate the result will be, but it is seldom necessary to use more than twenty lines.

Lengths of Pressure Lines in Fig. 6.

Crank End.	Head End.	Crank End.	Head End.
1 15/16	1 1/32	9/16	1/4
1 1/2	1 1/32	1/2	5/16
1 7/16	1 1/16	1/16	3/8
1 15/16	1 1/16	1/16	1/2
7/8	1 1/16	1/16	5/8
9/16	1 1/16	1/16	1 3/16
7/16	3/32	1/16	1 1/8
5/16	1/8	1/16	1 7/16
1/4	5/32	1/16	1 9/16
5/16	3/16	1/16	1 3/8

Examples:

1. From the measurements given in the diagram, Fig. 5, find the average length of the pressure lines.—Answer, 1.01 inch. The mean effective pressure is the average length of the pressure lines multiplied by the scale of the spring.

2. Find the mean effective pressure, supposing a 40-pound spring to have been used.—Answer, 40.4 pounds.

3. From the measurements given for Fig. 6, find the mean effective pressure for the crank end.—Answer, 0.42 inch.

4. From the same figure, find the mean effective pressure for the head end.—Answer, 0.46 inch.

5. We have now for Fig. 6 the average pressure for the crank end, and the average pressure for the head end. The average of these two results is the average pressure for a double stroke. Find this average.—Answer, 0.44 inch.

6. The scale of the indicator used for Fig. 6 is 40. Find the mean effective pressure.—Answer, 17.6 pounds. It will be noticed that in Fig. 6 twenty pressure lines are used. This gives a closer average than ten lines.

7. Fig. 5 is the indicator card for an engine having a piston of 11 inches diameter. What is the area of the piston?—Answer, 95 square inches.

8. What is the average total pressure on the piston of Example 7? The mean effective pressure has been found in Example 2. This is the average pressure per square inch. The average total pressure is the mean effective pressure multiplied by the area of the piston. Assume that pressures for head end and crank end are the same.—Answer, 3,838 pounds.

9. Fig. 6 is the indicator card for an engine having a piston of 24 inches diameter. What is the area of the piston?— Answer, 452.4 square inches.

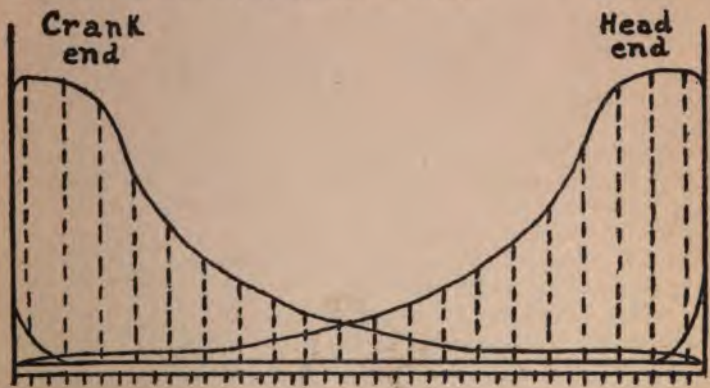


FIG. 6.

INDICATOR DIAGRAM TAKEN FROM A CORLISS ENGINE.

10. What is the average total pressure on the piston of Example 7?—Answer, 7,962.3 pounds.

Now, to find the amount of work done by the steam in one stroke of the piston, we must multiply the average total pressure in pounds by the distance in feet which the piston travels in one stroke. For example,

suppose the average total pressure is 6,000 pounds and the length of the stroke is 16 inches. Now, in one double stroke, that is during one revolution of the fly-wheel, the piston travels twice the length of the s' oke, or 32 inches, which equals 2⅔ feet. Multiplying 6,000 by 2⅔ we have 16,000. The work done is 16,000 foot-pounds.

11. If the piston stroke for Fig. 5 is 16 inches, how many foot-pounds of work are done in one revolution of the fly-wheel, that is in a double stroke of the piston? Take the average total pressure as found in Example 8. —Answer, 10,234 foot-pounds.

12. The piston stroke for Fig. 6 is 42 inches. How much work is done in one revolution of the fly-wheel? —Answer, 55,736 foot-pounds.

To find the work done in one minute we must multiply the work done in one revolution of the fly-wheel by the number of revolutions per minute.

13. How much work is done in one minute by the engine of Example 11, if the fly-wheel makes 110 revolutions per minute?—Answer, 1,125,740 foot-pounds per minute. To find the horse-power divide foot-pounds per minute by 33,000.

14. Find the horse-power of the engine of Example 13.—Answer, 34.1 horse-power.

15. How much work is done in one minute by the engine of Example 12, the fly-wheel making 78 r. p. m. —Answer, 4,347,408 foot-pounds per minute.

16. Find the horse-power of the engine of Example 15.—Answer 131.7 horse-power.

We can sum up the method of finding the horse-power of an engine in the following rule:

$$\text{Horse-power} = \frac{\text{mean effective pressure} \times \text{area of piston} \times \text{stroke in feet} \times 2 \times \text{r. p. m.}}{33000}$$

17. Using this rule, find the horse-power of an engine if the mean effective pressure is 15 pounds,

diameter of piston 22 inches, stroke 40 inches, and r. p. m. 80.—Answer, 92.1 horse-power.

18. If 25 per cent of the power of the engine in Example 17 is lost in transmission, that is if its efficiency is 75 per cent, what power is transmitted to the machinery that is driven by the engine?—Answer, 69 horse-power.

Finding the Capacity of Tanks and Boilers.

Suppose we have a circular tank 8 feet in diameter and 6 feet in height, how many gallons of water will it hold? A gallon equals 231 cubic inches. We must, therefore, find how many cubic inches there are in the tank and divide this number by 231.

To find the number of cubic inches, we must first find the number of square inches in the bottom of the tank and multiply this by the height of the tank. The bottom of this tank is a circle. Its diameter is 8 feet. Its radius is 4 feet. To find the area of a circle we square the radius, that is, multiply the radius by itself and multiply the result by 3.1416.

4 feet = 48 inches.

$$\begin{array}{r} 48 \\ 48 \\ \hline 384 \\ 192 \\ \hline 2{,}304 \end{array}$$

The square of the radius is 2,304 square inches.

$$\begin{array}{r} 3.1416 \\ 2{,}304 \\ \hline 12.5664 \\ 942.48 \\ 6{,}283.2 \\ \hline 7{,}238.2464 \end{array}$$

There are 7,238 square inches in the bottom of the tank. The decimal is less than ¼ of a square inch, and is so small that it may be dropped.

Multiplying the area of the bottom by the height of the tank, 72 inches, we have

$$\begin{array}{r} 7{,}238 \\ 72 \\ \hline 14{,}476 \\ 50{,}666 \\ \hline 521{,}136 \text{ cubic inches.} \end{array}$$

Dividing by 231 to reduce to gallons, we have

$$\begin{array}{r} 231 \overline{)\,521{,}136\,(\,2{,}256} \\ 462 \\ \hline 591 \\ 462 \\ \hline 1{,}293 \\ 1{,}155 \\ \hline 1{,}386 \\ 1{,}386 \\ \hline \end{array}$$

The tank holds 2,256 gallons.

The same method is used in finding the capacity of a boiler. Find the area of the end of the boiler by the rule for area of a circle and multiply this area by the length of the boiler. For a fire-tube boiler, first find the capacity of the boiler as though it had no tubes, then find the capacity of the tubes and subtract from the capacity of the boiler. To find capacity of tubes multiply the area of the end of a tube by its length and this product by the number of tubes.

Rule for Capacity of Circular Tank or Boiler:

Capacity in gallons equals area of end in square inches, times length or height in inches divided by 231.

Examples:

1. There is a water tank on the roof 10 feet in diameter and 8 feet in height. How many gallons does it hold?—Answer, 4,700 gallons.

2. If the diameter were one-half as great, that is, five feet, and the height 8 feet, would the tank hold one-half as much?—Answer, no; it would hold one-fourth as much.

3. How many cubic feet are there in a circular tank 9 feet in diameter and 6 feet in height.—Answer, 381.6 cubic feet.

4. A boiler is 20 feet long and 66 inches in diameter. How many gallons of water would fill it if there were no tubes?—Answer, 3,554½ gallons.

5. A water-tube boiler has 66 tubes. Each tube is 3 inches in diameter and 18 feet long. How many cubic inches of water does each tube hold? How many gallons does the boiler hold?—Answer, each tube holds 1,526 cubic inches. The boiler holds 436 gallons.

6. A fire-tube boiler is 60 inches in diameter and 20 feet long. How many gallons of water will fill it if one-half the space is taken up by the tubes?—Answer, 1,468.8 gallons.

In the preceding examples we have supposed, for the sake of simplicity, that the boiler is completely filled. This is not done in practice. To find the number of gallons actually in the boiler, we must find the contents of the space above the water and subtract this result from the entire contents of the boiler. The contents of the space above the water may be found

as follows: Let Fig. 7 represent the end of the boiler, AC the water level, O the center of the circle. Find what fraction the arc ABC is of the entire circumfer-

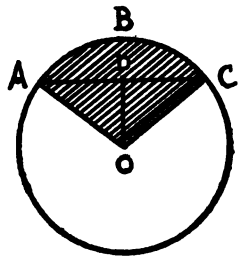


FIG. 7.

ence. The shaded area in the figure is the same fraction of the whole circle that ABC is of the circumference.

For example:

If ABC were one-third of the circumference, the shaded area would be one-third of the area of the circle. But the triangle AOC is below the water level. We must subtract the area of this triangle from the shaded area. DO is the height of water level above the center of the boiler, and AC the width of the boiler at the water level. Both are easily measured. One-half the length of AC times DO is the area of the triangle. Subtracting the area of the triangle from the entire shaded area gives the area of the segment above the water level. Multiplying this area by the length of the boiler gives the capacity to be subtracted from the entire capacity of the boiler.

7. Suppose the diameter of the boiler is 60 inches, water level is 15 inches above the center, width of at water level is 52 inches, length of arc ABC

is 63 inches, length of boiler 20 feet. What is the capacity in cubic inches of the space above the water?

Solution:

Area of end of boiler is 2,827 square inches.

Circumference of boiler is 189 inches, nearly.

Arc ABC is one-third of circumference.

Area of AOCB is one-third of 2,827, or 942 square inches.

Area of triangle ADC is one-half of 52×15, or 390.

Area of segment ABCD is $942 - 390 = 552$ square inches.

Length of boiler is 240 inches.

$552 \times 240 = 132{,}480$ cubic inches.

If a tank is square instead of circular, we measure the length, breadth and height in inches, multiply these numbers together and divide by 231 to find the number of gallons.

8. How many gallons will a tank 8 by 10 by 6 feet hold? — Answer, 3,590 gallons.

A rough and ready method of finding the number of gallons a tank or boiler will hold is to find the number of cubic feet and multiply by 7.5. This gives a result which is nearly correct, for there are nearly 7½ gallons in a cubic foot.

9. Find roughly the number of gallons in a tank 6 by 8 by 8 feet.— Answer, 2,880 gallons.

10. Find roughly the number of gallons in a circular tank, the diameter being 8 feet and the height 8 feet. Use 3⅛ instead of 3.1416 in finding area.— Answer, 3,017 gallons.

For a tank with sloping sides we shall give only a rough and ready method which gives a result slightly too large, but near enough for most practical purposes.

The top and bottom of such a tank have different areas. Find these two areas, add them and divide by 2. Multiply this result by the height of the tank.

11. Find roughly the capacity of a circular tank with sloping sides, the diameter of the top being 8 feet, the diameter of the bottom 10 feet, and the height 8 feet.— Answer, 3,865 gallons.

Finding Cost of Power.

To find the cost of operating his plant is one of the principal duties of the engineer. This is really a simple matter, as we shall see. We shall explain methods of finding both the cost of steam power and the cost of electrical power.

The cost depends on the rate at which fuel is consumed per horse-power, and the amount of coal per horse-power depends on the number of pounds of coal needed to evaporate one pound of water, and the number of pounds of water that must be changed to steam every hour for one horse-power. The engineer should know how to find the number of pounds of steam required per hour per horse-power. We shall explain this first.

To Find Number of Pounds of Steam Per Hour.

By this we mean, not pounds of steam pressure, but the number of pounds of water that must be changed into steam every hour.

In Table I is given the weight in pounds of a cubic foot of steam at different pressures. We take from the indicator-card the pressure at " release." The cylinder is practically filled with steam at this pressure. In reality, the pressure taken should be that which the pressure would be if there were no compression. This would be the pressure given by extending the expansion line until it cuts the vertical line at the end of the card as shown in Fig. 8. It can be seen from the figure that this is practically the same as the pressure at release.

TABLE No. I.

WEIGHT OF CUBIC FOOT OF STEAM AT DIFFERENT PRESSURES, TOGETHER WITH HEAT UNITS.

Gauge Pressure, Lbs. per Sq. In.	Absolute Pressure, Lbs. per Sq. In.	Temperature, Fahrenheit.	Total Heat above 32° F		Latent Heat, $L = H - A$ Heat-Units.	Volume, Cu. Ft. of 1 Lb. of Steam.	Weight of 1 Cu. Ft. Steam, Lb.	Entropy of the Water.	Entropy of Evaporation.
			In the Water A Heat-Units.	In the Steam H Heat-Units.					
81 3	96	324 9	295 3	1185 6	890 3	4 60	0.2172	0 4704	1 1348
82 3	97	325 6	296 1	1185 8	889 7	4 56	0.2193	0.4714	1 1330
83 3	98	326 4	296 8	1186 0	889.2	4 51	0 2215	0.4724	1 1312
84 3	99	327 1	297 6	1186 2	888 6	4 47	0 2237	0.4733	1 1295
85 3	100	327 8	298 3	1186 3	888.0	4.429	0 2258	0.4743	1 1277
87 3	102	329 3	299 8	1186.7	886 9	4.347	0 2300	0.4762	1 1242
89 3	104	330.7	301 3	1187 0	885 8	4 268	0 2343	0 4780	1 1208
91 3	106	332 0	302.7	1187.4	884 7	4.192	0 2336	0.4798	1 1174
93 3	108	333 4	304 1	1187.7	883 6	4.118	0.2429	0 4816	1 1141
95 3	110	334 8	305 5	1188.0	882 5	4.047	0 2472	0 4834	1 1108
97 3	112	336 1	306 9	1188.4	881 4	3.978	0 2514	0 4852	1 1076
99 3	114	337 4	308 3	1188 7	880.4	3.912	0 2556	0 4869	1 1045
101 3	116	338 7	309 6	1189 0	879.3	3.848	0 2599	0 4886	1 1014
103 3	118	340 0	311 0	1189 3	878 3	3.786	0 2641	0.4903	1.0984
105 3	120	341 3	312 3	1189 6	877.2	3.726	0 2683	0 4919	1 0954
107 3	122	342 5	313 6	1189 8	876.2	3.668	0.2726	0.4935	1 0924
109 3	124	343 8	314 9	1190 1	875.2	3.611	0.2769	0 4951	1.0895
111 3	126	345 0	316 2	1190 4	874.2	3.556	0 2812	0 4967	1 0865
113 3	128	346 2	317 4	1190 7	873.3	3 504	0.2854	0 4982	1.0837
115 3	130	347 4	318 6	1191 0	872.3	3 452	0.2897	0.4998	1.0809
117 3	132	348 5	319 9	1191 2	871 3	3 402	0 2939	0 5013	1.0782
119 3	134	349 7	321 1	1191 5	870 4	3 354	0 2981	0.5028	1.0755
121 3	136	350 8	322 3	1191.7	869 4	3 308	0.3023	0.5043	1.0728
123 3	138	352.0	323 4	1192 0	868.5	3 263	0.3065	0.5057	1.0702
125 3	140	353 1	324 6	1192 2	867 6	3 219	0 3107	0 5072	1.0675
127 3	142	354 2	325 8	1192 5	866 7	3 175	0 3150	0 5086	1.0649
129 3	144	355 3	326 9	1192 7	865 8	3 133	0 3192	0 5100	1.0624
131 3	146	356 3	328 0	1192 9	864 9	3 092	0 3234	0.5114	1.0599
133 3	148	357 4	329 1	1193 2	864 0	3 052	0 3276	0 5128	1.0574
135 3	150	358 5	330 2	1193 4	863 2	3 012	0 3320	0 5142	1.0550
137 3	152	359 5	331 4	1193 6	862 3	2 974	0 3362	0 5155	1.0525
139 3	154	360 5	332 4	1193 8	861 4	2 938	0 3404	0 5169	1 0501
141 3	156	361 6	333 5	1194 1	860 6	2 902	0 3446	0 5182	1.0477
143 3	158	362 6	334 6	1194 3	859 7	2 868	0 3488	0 5195	1.0454
145 3	160	363 6	335 6	1194 5	858 8	2 834	0 3529	0 5208	1.0431
147 3	162	364 6	336 7	1194 7	858 0	2 801	0 3570	0.5220	1.0409
149 3	164	365 6	337 7	1194 9	857 2	2 769	0 3612	0 5233	1.0387
151 3	166	366 5	338 7	1195 1	856 4	2 737	0 3654	0.5245	1.0365
153 3	168	367 5	339 7	1195 3	855 5	2 706	0.3696	0.5257	1.0343
155 3	170	368 5	340 7	1195 4	854 7	2.675	0 3738	0.5269	1.0321
157 3	172	369 4	341 7	1195 6	853 9	2 645	0 3780	0.5281	1.0300
159 3	174	370 4	342 7	1195 8	853 1	2 616	0 3822	0.5293	1.0278
161 3	176	371 3	343 7	1196 0	852 3	2 588	0 3864	0.5305	1.0257
163 3	178	372.2	344 7	1196 2	851 5	2.560	0 3906	0 5317	1.0235
165 3	180	373 1	345.6	1196 4	850 8	2 533	0 3948	0 5328	1.0215
167.3	182	374 0	346 6	1196 6	850 0	2.507	0.3989	0 5339	1.0195
169.3	184	374 9	347 6	1196 8	849 2	2.481	0 4031	0.5351	1.0174
171.3	186	375.8	348.5	1196.9	848.4	2.455	0.4073	0.5362	1.0154

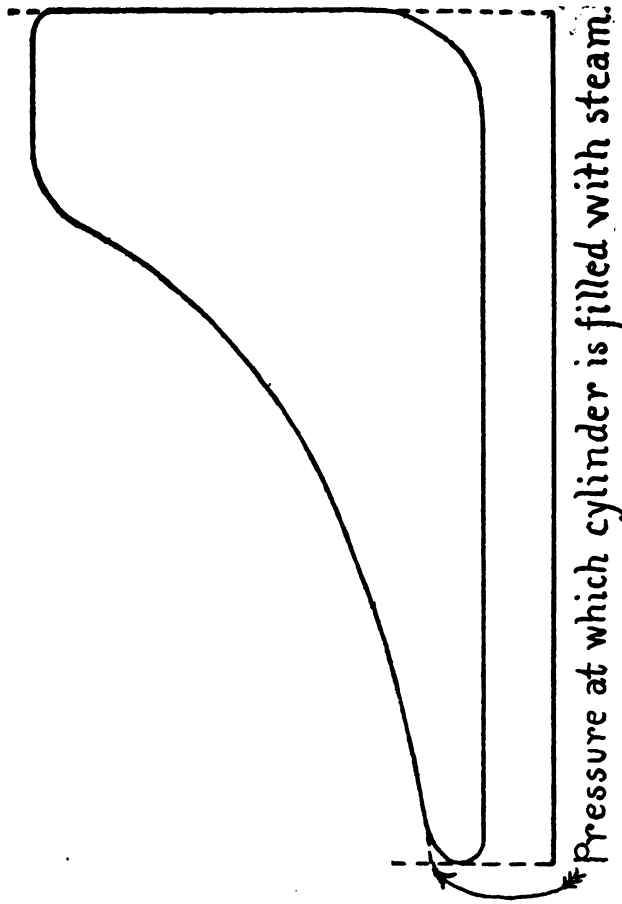


FIG. 8.

Now from the table we find the weight of a cubic foot of steam at this pressure. Multiplying this weight by the number of cubic feet in the cylinder gives the weight of steam, also the weight of water used, for each stroke of the piston. Multiplying this by the number of strokes per minute gives weight of water used per minute. Multiplying this result by 60 gives weight of water changed to steam in one hour. More water than this may be evaporated in the boiler, but the excess is wasted. It is not used in doing work in the engine.

Having found the rate of consumption of water, we must next know the amount of coal used in evaporating one pound of water. One pound of coal will evaporate from five to sixteen pounds of water into steam from and at 212° F., depending on the character of the coal.

The unit of heat used in engineering calculations is the **British thermal unit,** usually written B. t. u. This unit is the amount of heat required to raise the temperature of one pound of water one degree Fahrenheit; or, more strictly defined, it is the amount of heat required to raise one pound of water from 39° F. to 40° F.

The unit of evaporation is one pound of water at 212° F. evaporated into dry steam of the same temperature.

The unit of evaporation is equivalent to 965.7 B. t. u. That is, it requires 965.7 B. t. u. of heat to evaporate one pound of water under the conditions named; 966 is near enough for most calculations.

Evaporative Test.

We shall explain here fully all of the calculations that must be made in an evaporative test. For methods of working in making the test, some good book on engineering practice should be consulted. We shall

explain first the different parts of the test, and then show how these are combined into a complete test.

The purpose of this test is to find the cost of producing the steam and the efficiency of the boilers.

We may divide the calculations for finding cost into the following parts:

1. Finding weight of coal and weight of combustible used.
2. Finding weight of water evaporated.
3. Finding pounds of water evaporated per pound of combustible.
4. Finding equivalent weight of water evaporated into steam from and at 212° F.
5. Finding cost of evaporating 1,000 pounds of water.

Other data for determining the efficiency of the boilers are usually taken while the test is being made. These are explained farther on.

Finding Weight of Coal and Weight of Combustible Used.

Total weight of coal shoveled into the furnace during the test must be known. Weight of coal, clinkers and ashes left in furnace and ash pit at end of test must be subtracted from total weight of coal. Weight of moisture in the coal must also be subtracted from weight of coal.

To find the percentage of moisture in the coal, weigh a sample of the coal as delivered, then dry it thoroughly and weigh it again. The second weight subtracted from the first gives the weight of moisture in this sample of coal. The weight of moisture divided by the weight of coal as first taken gives the weight of moisture in one pound of coal. This last weight, multiplied by 100, gives the weight of moisture in 100 pounds of coal, and, if weight of moisture is also in pounds, this is the same as the percentage of moisture

Typical Proximate Analyses and Heating Values for Various American Coals.

	Fixed Carbon per cent	Volatile Matter per cent	Moisture per cent	Ash per cent	Calorific Value per pound B. t. u.
Anthracite	86.40	3.67	3.71	6.22	13420
Semi-bituminous [Pocahontas]	75.63	19.83	0.50	4.04	14218
Bituminous [Illinois]	44.14	36.35	6.66	12.86	11954
Lignite	29.55	29.28	36.13	5.04	7326

TABLE NO. II.

HEATING VALUE OF COAL.

for percentage as we have seen, is the number of parts in one hundred.

Example:

A sample of coal weighs 6 pounds before drying, and 4 pounds after drying. What is the percentage of moisture?

Solution:

The water in this sample of coal weighs 2 pounds. Dividing 2 by 6, we find that there is ⅓ of a pound of moisture in every pound of coal. Multiplying ⅓ by 100, we find that there are 33⅓ pounds of moisture in 100 pounds of coal. This equals 33⅓ per cent.

Examples to be solved:

1. Find the percentage of moisture in coal when a sample of the coal as delivered weighs 5 pounds, and when dried weighs 4 pounds.— Answer, 20 per cent.
2. Find the percentage of moisture in coal if a sample weighs 7½ pounds before drying and 5¼ pounds after drying.— Answer, 30 per cent.
3. In an evaporative test, 12,448 pounds of coal were used. How many pounds of dry coal were used if the coal contained 12 per cent of moisture? Hint: If 12 per cent was moisture, then 88 per cent was dry coal.— Answer, 10,954.24 pounds.
4. In 12,448 pounds of coal used in an evaporative test there was found to be 834 pounds of unburned coal and refuse, consisting of ashes and clinkers, at the end of the test. There was 12 per cent of moisture as in Example 3. How many pounds of combustible were used in the test? Deduct for moisture and refuse and unburned coal.— Answer, deducting for moisture leaves 10,954.24 pounds dry coal shoveled into the furnace. Deducting from this the weight of refuse and unburned coal leaves 10,120.24 pounds of combustible consumed in the furnace.

To calculate the heat value of the coal it is necessary to separate the ashes and clinkers from the unburned coal and find the per cent of ashes and clinkers.

5. Suppose in the above example there were 200 pounds of unburned coal. What is the per cent of refuse?

Solution:

Deducting weight of unburned coal, 200 pounds, from total weight of unburned coal and refuse, leaves 634 pounds of refuse. Deducting weight of unburned coal from total weight of coal leaves 12,248 pounds of coal burned.

$$
\begin{array}{r}
12{,}248\overline{)634.00(.0517} \\
61240 \\
\hline
21600 \\
12248 \\
\hline
93520 \\
85736 \\
\hline
7784
\end{array}
$$

Answer, 5.17 per cent of refuse.

6. In an evaporative test, 11,000 pounds of coal are used. There were 10 per cent of moisture and 950 pounds of ashes, clinkers and unburned coal. There were 300 pounds of unburned coal. How many pounds of dry coal were used? How many pounds of combustible were consumed in the furnace? What was the per cent of ashes and clinkers in the coal?

Answer, 9,900 pounds dry coal; 8,950 pounds combustible; 6.07 per cent ashes and clinkers.

Finding Weight of Water Evaporated.

The weight of water apparently evaporated is usually not the same as the weight of water actually evaporated, for the reason that some water usually goes over with the steam. In a complete test it is necessary to find the per cent of moisture in the steam. This is done by the calorimeter test. The best method of making this test is with the throttling calorimeter. One form of throttling calorimeter is shown in Fig. 9.

The percentage of moisture is found as follows:

Find from table the total heat in a pound of steam at the pressure in the pipe. Subtract from this the total heat in a pound of steam at the pressure in the calorimeter. Call this the first answer. Now subtract the temperature of the boiling point at the pressure in the calorimeter from the temperature in the calorimeter. This gives degrees of superheating in calorimeter. Multiply this result by 0.48. This gives the heat in a pound of steam in calorimeter due to superheating. Call this the second answer. Subtract the second answer from the first, divide by the latent heat in a pound of steam at the pressure in the pipe and multiply by 100. The last answer subtracted from 100 gives the quality of the steam. Quality of steam is the per cent of dry steam, or 100 minus per cent of moisture. The pressure in the calorimeter may be taken as equal to the atmospheric pressure, and the boiling point in calorimeter as 212°. Table V gives per cent of moisture in steam for different pressures and degrees of superheat in calorimeter.

The Barrel Calorimeter.

If a throttling calorimeter is not at hand, a barrel calorimeter may be used. To make this test, a known weight of clean, cold water is taken and its temperature noted. A pipe is tapped in such a way as to get

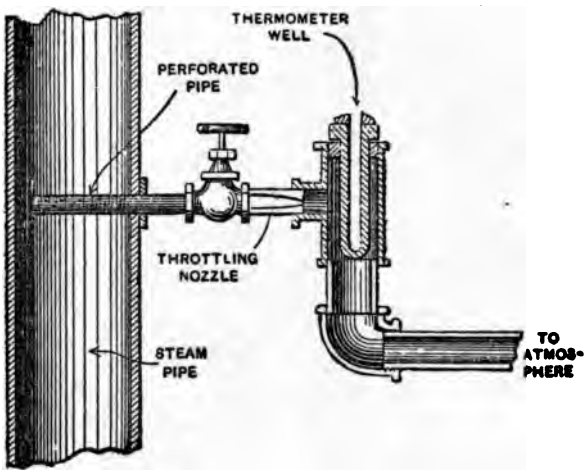


FIG. 9.

A THROTTLING COLORIMETER.

steam of the same quality as that generated by the boiler. This steam is then blown into the cold water until its temperature is raised to about 125° F., and the temperature and weight again taken. The calculation is then made as in the following example:

Weight of tub used as calorimeter.....6 pounds.
Weight of tub with water.....36 pounds.
Weight of water.....30 pounds.
Temperature of water.....55° F.
Temperature of water after blowing in steam..130° F.
Rise of temperature.....75° F.
Weight of tub and water after blowing in steam .....38.15 pounds.
Weight of steam condensed..... 2.15 pounds.
Heat received by cold water (30 pounds, warmed 75°), $30 \times 75 =$.....2,250 B. t. u.

2,250 B. t. u. is therefore the heat given up by the steam in condensing and cooling to 130°.

Heat given up by 1 pound of steam in condensing and cooling to 130° $= \frac{2,250}{2.15} =$ 1,046.5 B. t. u.

To find the total heat in 1 pound of this steam, add the heat it would give up if cooled from 130° to 0°, or 130 B. t. u.

Total heat in 1 pound of steam from boiler, 1,046.5 + 130 = 1,176.5 B. t. u.

Total heat of 1 pound of saturated steam, counted from 0° F., at pressure carried on boiler (70 pounds), from table = 1,206.2.

Subtracting total heat in 1 pound of steam as it comes from the boiler from the total heat in 1 pound of saturated steam, gives the additional heat that would have been present if the water suspended in 1 pound of the steam had been evaporated. This is the heat it would have taken to evaporate the water suspended

English Unit.

Absolute Pressure Pounds per Sq. Inch p	Temperature Degrees Fahr. t	Heat of the Liquid Above 32 Deg. Fahr. q	Total Heat λ	Heat of Vaporization r	Volume in Cu. Ft. of 1 Pound s
5	162.34	130.7	1131.5	1000.8	73.22
10	193.25	161.9	1140.9	979.0	38.16
15	213.03	181.8	1146.9	965.1	26.15
20	227.95	196.9	1151.5	954.6	19.91
25	240.04	209.1	1155.1	946.0	16.13
30	250.27	219.4	1158.3	938.9	13.59
35	259.19	228.4	1161.0	932.6	11.75
40	267.13	236.4	1163.4	927.0	10 87
45	274.29	243.6	1165.6	922.0	9.287
50	280.85	250.2	1167.6	917.4	8.414
55	286.89	256.3	1169.4	913.1	7.696
60	292.51	261.9	1171.2	909.3	7.096
65	297.77	267.2	1172.7	905.5	6.583
70	302.71	272.2	1174.3	902.1	6.144
75	307.38	276.9	1175.7	898.8	5.762
80	311.80	281.4	1177.0	895.6	5.425
85	316.02	285.8	1178.3	892.5	5.125
90	320.04	290.0	1179.6	889.6	4.858
95	323.89	294.0	1180.7	886.7	4.619
100	327.58	297.9	1181.9	884.0	4.403
105	331.13	301.6	1182.9	881.3	4.206
110	334.56	305.2	1184.0	878.8	4.026
115	337.86	308.7	1185.0	876.3	3.862
120	341.05	312.0	1186.0	874.0	3.711
125	344.13	315.2	1186.9	871.7	3.572
130	347.12	318.4	1187.8	869.4	3.444
135	350.03	321.4	1188.7	867.3	3.323
140	352.85	324.4	1189.5	865.1	3.212
145	355.59	327.2	1190.4	863.2	3.107
150	358.26	330.0	1191.2	861.2	3.011
155	360.86	332.7	1192.0	859.3	2.919
160	363.40	335.4	1192.8	857.4	2.833
165	365.88	338.0	1193.6	855.6	2.751
170	368.29	340.5	1194.3	853.8	2.676
175	370.65	343.0	1195.0	852.0	2.603
180	372.97	345.4	1195.7	850.3	2.535
185	375.23	347.8	1196.4	848.6	2.470
190	377.44	350.1	1197.1	847.0	2.408
195	379.61	352.4	1197.7	845.3	2.349
200	381.73	354.6	1198.4	843.8	2.294
205	383.82	356.8	1199.0	842.2	2.241
210	385.87	358.9	1199.6	840.7	2.190
215	387.88	361.0	1200.2	839.2	2.142
220	389.84	363.0	1200.8	837.8	2.096
225	391.79	365.1	1201.4	836.3	2.051
230	393.69	367.1	1202.0	834.9	2.009
235	395.56	369.0	1206.6	833.6	1.968
240	397.41	371.0	1203.2	832.2	1.928
245	399.21	372.8	1203.7.	830.9	1.891
250	400.99	374.7	1204.2	829.5	1.854

TABLE No. III.

PROPERTIES OF SATURATED STEAM.

WATER.

Water at Different Temperatures.

Freezing point at sea level	32 deg. Fahr.
Point of maximum density	39.1 " "
British standard for specific gravity	62 " "
Boiling point at sea level	212 " "
Weight per cu. ft. at 32 deg. Fahr	62.418 lbs.
" " 39.1 " "	62.425 "
" " 62 " "	62.355 "
" " 212 " "	59.760 "
Weight per cu. in at 32 " "	.03612 "
" " 39.1 " "	.036125 "
" " 62 " "	.03608 "
" " 212 " "	.03458 "

Boiling Point of Water.

Barometer at 31 inches	213.57°
" " 29 "	210.38°
" " 28 "	208.69°
" " 27 "	206.85°
" in vacuo	88°

TABLE No. IV.

BOILING POINT OF WATER AT DIFFERENT PRESSURES. ALSO AT DIFFERENT TEMPERATURES.

Determinations by Throttling Calorimeter.

$(t - t_0)$	Gauge-pressures.											
	5	10	20	30	40	50	60	70	75	80	85	90
	Per Cent of Moisture in Steam.											
0°	0.51	0.90	1.54	2.06	2.50	2.90	3.24	3.56	3.71	3.86	3.99	4.13
10°	0.01	0.39	1.02	1.54	1.97	2.36	2.71	3.02	3.17	3.32	3.45	3.58
20°	...	...	.51	1.02	1.45	1.83	2.17	2.48	2.63	2.77	2.90	3.03
30°	...	...	.00	.50	.92	1.30	1.64	1.94	2.09	2.23	2.35	2.49
40°	...	...	...	...	.39	.77	1.10	1.40	1.55	1.69	1.80	1.94
50°	...	...	...	...	...	.24	.57	.87	1.01	1.15	1.26	1.40
60°	...	...	...	...	...	...	.03	.33	.47	.60	.72	.85
70°	...	...	...	...	...	...	...	...	...	.06	.17	.31

$(t - t_0)$	Gauge-pressure.											
	100	110	120	130	140	150	160	170	180	190	200	250
	Per Cent of Moisture in Steam.											
0°	4.39	4.63	4.85	5.08	5.29	5.49	5.68	5.87	6.05	6.22	6.39	7.16
10°	3.84	4.08	4.29	4.52	4.73	4.93	5.12	5.30	5.48	5.65	5.82	6.58
20°	3.29	3.52	3.74	3.96	4.17	4.37	4.56	4.74	4.91	5.08	5.25	6.00
30°	2.74	2.97	3.18	3.41	3.61	3.80	3.99	4.17	4.34	4.51	4.67	5.41
40°	2.19	2.42	2.63	2.85	3.05	3.24	3.43	3.61	3.78	3.94	4.10	4.83
50°	1.64	1.87	2.08	2.29	2.49	2.68	2.87	3.04	3.21	3.37	3.53	4.25
60°	1.09	1.32	1.52	1.74	1.93	2.12	2.30	2.48	2.64	2.80	2.96	3.67
70°	.55	.77	.97	1.18	1.38	1.56	1.74	1.91	2.07	2.23	2.38	3.09
80°	.00	.22	.42	.63	.82	1.00	1.18	1.34	1.50	1.66	1.81	2.51
90°	...	...	...	.07	.26	.44	.61	.78	.94	1.09	1.24	1.93
100°	...	...	...	...	...	...	.05	.21	.37	.52	.67	1.34
110°	...	...	...	...	...	...	...	...	...	...	.10	.76

TABLE No. V.

PERCENTAGE OF MOISTURE OF STEAM FOR DIFFERENT PRESSURES AND DEGREES OF SUPERHEAT IN CALORIMETER.

in 1 pound of the steam. It is, therefore, the latent heat of the water which remains unevaporated in 1 pound of the steam. Dividing this result by the latent heat of steam at the boiler pressure, that is, by the heat required to completely evaporate 1 pound of steam at the boiler pressure, gives the weight of suspended water in 1 pound of the boiler steam. Multiplying this result by 100 gives the number of pounds of suspended water in 100 pounds of the boiler steam. This is the per cent of moisture.

To finish the calculation:

Latent heat of suspended water in 1 pound of boiler steam = 1,206.2 — 1,176.5 = 29.7 B. t. u.

Latent heat of 1 pound of saturated steam at boiler pressure, 900.9.

Weight of suspended water in 1 pound of boiler steam = $29.7 \div 900.9 = 0.032$ pound.

Weight of suspended water in 1 pound of boiler steam = $0.032 \times 100 = 3.2$ pounds.

Per cent of moisture in steam, 3.2.

Example:

If water apparently evaporated is 11,000 pounds, and the calorimeter test shows that the steam contains 3 per cent of moisture, how many pounds of water have been actually evaporated? — Answer, 10,670 pounds.

Finding Weight of Water Evaporated Per Pound of Combustible.

This is a simple matter after finding the weight of water actually evaporated and the number of pounds of combustible consumed in the test. Divide the weight of water evaporated by the weight of combustible.

Example:

Pounds of water actually evaporated, 81,145. Pounds of combustible, 8,280. How many pounds of water were evaporated per pound of combustible? — Answer, 9.8 pounds.

Finding Equivalent Weight of Water Evaporated into Steam from and at 212° F.

It would be impossible to compare different boiler tests without reducing them to standard conditions. The standard conditions of evaporation are water at 212° F. evaporated into steam at 212° F.

To evaporate 1 pound of water at 212° F. into steam at 212° F. requires 965.7 British thermal units of heat. Hence a pound of steam at 212° contains 965.7 B. t. u. more of heat than is contained in a pound of water at 212°. A pound of water at 212° contains 212 B. t. u., reckoned from 0°.

Therefore, the total heat of a pound of steam at 212°, reckoned from zero, is 965.7 + 212, or 1,177.7 B. t. u.

In the evaporation test the temperature is higher than 212°, and the pressure higher than atmospheric pressure. We must find how much water at 212° would have been evaporated into steam at 212° by the amount of heat that was actually used. This can be found as follows: Multiply the total weight of water evaporated under the actual conditions by the total heat of steam under the actual conditions minus heat of feed-water, and divide by 965.7.

The same rule can be put in the following form, which may make it more clear. The water that would have been evaporated from and at 212° has the same ratio to the water actually evaporated that the heat of steam under the actual conditions has to the heat of steam at 212°.

To simplify this calculation, a table of factors of evaporation (Table VI) is given. Find from the table the factor of evaporation for the boiler pressure and temperature of feed-water as used in the test. Multiply the weight of water evaporated under the actual conditions by this factor. The result will be the equivalent weight of water at 212° into steam at 212°.

The usual method is to find first the weight of water evaporated per pound of combustible under the actual conditions and multiply this number by the factor of evaporation. The result is the weight of water at 212° into steam at 212° per pound of combustible.

Examples:

1. If 11 pounds of water per pound of combustible are evaporated under actual conditions, and the factor of evaporation is 1.06, what is the equivalent weight of water at 212° into steam at 212° per pound of combustible? — Answer, 11.66 pounds.

2. If 11.5 pounds of water per pound of combustible are evaporated under the actual conditions, and the temperature of feed-water is 200° F. and boiler pressure is 75 pounds, what is the weight of water at 212° into steam at 212° per pound of combustible? — Answer, 12.03 pounds.

Finding Cost of Evaporating 1,000 Pounds of Water.

Having found the weight of water evaporated into steam from and at 212° per pound of combustible, we can find the cost of evaporating 1,000 pounds of water, standard conditions, as follows: (1) Find weight of combustible required to evaporate 1,000 pounds of water; (2) find weight of coal required to furnish this weight of combustible; (3) find cost of this weight of coal.

Temp. of Feed Water Deg. Fahr.	Steam Pressure by Gauge.																								
	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250	260	270	280	290	300
32	1.214	1.216	1.220	1.222	1.225	1.227	1.229	1.231	1.232	1.234	1.236	1.237	1.239	1.240	1.241	1.243	1.244	1.245	1.246	1.247	1.248	1.250	1.251	1.252	1.253	1.254
40	1.206	1.209	1.212	1.214	1.216	1.219	1.220	1.222	1.224	1.226	1.227	1.229	1.230	1.232	1.233	1.234	1.236	1.237	1.238	1.239	1.240	1.241	1.242	1.243	1.244	1.245
50	1.195	1.197	1.201	1.204	1.206	1.208	1.210	1.212	1.214	1.215	1.217	1.218	1.220	1.221	1.223	1.224	1.225	1.226	1.228	1.229	1.230	1.231	1.232	1.233	1.234	1.235
60	1.185	1.188	1.191	1.193	1.196	1.198	1.200	1.202	1.203	1.205	1.207	1.208	1.210	1.211	1.212	1.214	1.215	1.216	1.217	1.218	1.219	1.220	1.221	1.222	1.223	1.224
70	1.175	1.178	1.180	1.183	1.185	1.187	1.189	1.191	1.193	1.194	1.196	1.197	1.199	1.200	1.202	1.203	1.205	1.206	1.207	1.208	1.209	1.210	1.211	1.212	1.213	1.214
80	1.164	1.167	1.170	1.173	1.175	1.177	1.179	1.181	1.183	1.184	1.186	1.187	1.189	1.190	1.192	1.193	1.194	1.195	1.196	1.198	1.199	1.200	1.201	1.202	1.203	1.204
90	1.154	1.157	1.160	1.162	1.165	1.167	1.169	1.170	1.172	1.174	1.176	1.177	1.179	1.180	1.181	1.183	1.184	1.185	1.186	1.187	1.188	1.189	1.190	1.191	1.192	1.193
100	1.144	1.147	1.150	1.152	1.154	1.156	1.158	1.160	1.162	1.164	1.165	1.167	1.168	1.170	1.171	1.172	1.174	1.175	1.176	1.177	1.178	1.179	1.180	1.181	1.182	1.183
110	1.133	1.136	1.139	1.142	1.144	1.146	1.148	1.150	1.152	1.153	1.155	1.156	1.158	1.159	1.160	1.162	1.163	1.164	1.166	1.167	1.168	1.169	1.170	1.171	1.172	1.173
120	1.123	1.126	1.129	1.131	1.133	1.136	1.138	1.140	1.141	1.143	1.145	1.146	1.147	1.149	1.150	1.151	1.153	1.154	1.155	1.156	1.157	1.158	1.159	1.160	1.161	1.162
130	1.113	1.116	1.118	1.121	1.123	1.125	1.127	1.129	1.130	1.132	1.134	1.136	1.137	1.138	1.140	1.141	1.142	1.144	1.145	1.146	1.147	1.148	1.149	1.150	1.151	1.152
140	1.102	1.105	1.108	1.110	1.113	1.115	1.117	1.119	1.120	1.122	1.124	1.125	1.127	1.128	1.129	1.131	1.132	1.133	1.134	1.135	1.136	1.137	1.138	1.139	1.140	1.141
150	1.091	1.095	1.098	1.100	1.102	1.104	1.106	1.108	1.110	1.111	1.113	1.115	1.116	1.118	1.119	1.120	1.121	1.123	1.124	1.125	1.126	1.127	1.128	1.129	1.130	1.131
160	1.081	1.084	1.087	1.090	1.092	1.094	1.096	1.098	1.100	1.101	1.103	1.104	1.106	1.107	1.108	1.110	1.111	1.112	1.113	1.115	1.116	1.117	1.118	1.119	1.120	1.121
170	1.070	1.074	1.077	1.079	1.081	1.083	1.085	1.087	1.089	1.091	1.092	1.094	1.095	1.097	1.098	1.099	1.101	1.102	1.103	1.104	1.105	1.106	1.107	1.108	1.109	1.110
180	1.060	1.063	1.066	1.069	1.071	1.073	1.075	1.077	1.079	1.080	1.082	1.083	1.085	1.086	1.088	1.089	1.090	1.091	1.093	1.094	1.095	1.096	1.097	1.098	1.099	1.100
190	1.050	1.053	1.056	1.058	1.060	1.063	1.065	1.066	1.068	1.070	1.071	1.073	1.074	1.076	1.077	1.078	1.080	1.081	1.082	1.083	1.084	1.085	1.086	1.087	1.088	1.089
200	1.039	1.043	1.045	1.048	1.050	1.052	1.054	1.056	1.058	1.059	1.061	1.063	1.064	1.065	1.067	1.068	1.069	1.071	1.072	1.073	1.074	1.075	1.076	1.077	1.078	1.079
210	1.029	1.032	1.035	1.037	1.040	1.042	1.044	1.046	1.047	1.049	1.051	1.052	1.053	1.055	1.056	1.057	1.059	1.060	1.061	1.062	1.063	1.064	1.065	1.066	1.067	1.068

Table No. VI.

FACTORS OF EVAPORATION.

Example:

Water evaporated from and at 212° per pound of combustible, 7.48 pounds. Per cent of refuse in coal, 6.71. Per cent of moisture in coal (lignite), 39.76. Cost of coal per ton, delivered, $1.70. Find cost of evaporating 1,000 pounds of water.

Solution:

To find weight of combustible required to evaporate 1,000 pounds of water, divide 1,000 by 7.48.

$1{,}000 \div 7.48 = 133.69$ pounds combustible to evaporate 1,000 pounds of water from and at 212°.

Total per cent moisture and refuse in coal, 46.47.

Per cent combustible in coal, $100 - 46.47 = 53.53$.

Coal required to furnish 133.69 pounds combustible, $133.69 \div .5353 = 249.7$ pounds.

Cost of 249.7 pounds coal at $1.70 a ton, $\frac{\$1.70}{2{,}000} \times 249.7 = \$0.212$.

Hence it cost 21.2 cents to evaporate 1,000 pounds of water from and at 212° F.

Heating Value of Coal and Efficiency of Boilers.

The total amount of heat in a pound of coal is measured by a coal calorimeter. Table VII gives the heating value of a number of different grades of coal. The number of heat units absorbed in the water in the boiler for a pound of coal can be found from the evaporative test. This number divided by the number of heat units in a pound of the coal used gives the per cent of combined efficiency of the boilers, furnaces and grates.

Examples:

1. A test shows that 1 pound of coal as fired evaporates 4 pounds of water, and it requires 1,024 B. t. u. of heat to raise 1 pound of water from the temperature of the feed-water to steam at boiler pressure. The coal

contains 6,029 B. t. u. per pound. What is the combined efficiency of boiler, furnace and grates?

Solution:

1,024 × 4 = 4,096 B. t. u. absorbed in water in boiler per pound of coal burned.

4,096 ÷ 6,029 = 0.68.

Hence efficiency is 68 per cent; that is, the water absorbs 68 per cent of the heat that is in the coal.

2. If the coal used in a test has 11,290 B. t. u. per pound, and 1 pound of coal evaporates 7.3 pounds of water from and at 212°, and the temperature of the feed-water is 200°, what is the efficiency of boilers, furnaces and grates?

Hint: Heat of steam at 212° from 200° is 965.7 + 12 = 977.7 B. t. u. This result is higher than can be obtained in practice.— Answer, 86.5 per cent.

Comparative Value of Fuels.

The value of fuel depends on the number of heat units it contains. For example: North Dakota lignite coal was purchased at $1.70 a ton, delivered. In another plant coal was purchased at $2.15 a ton. The first coal contained only 6,029 B. t. u. per pound. The second contained 11,290 B. t. u. per pound. Which coal was the cheaper? It is clear that the second coal contained nearly twice as many heat units per pound as the first, and the cost was less in proportion to amount of heat. The first coal supplied 12,058,000 heat units per ton — 12,058,000 heat units for $1.70. The second coal supplied 22,580,000 heat units per ton, or this amount of heat for $2.15. If we take $\frac{11,290}{6,029} \times \$1.70$ we get $3.18; that is, if we buy the first coal at $1.70 we pay $3.18 for the same amount of heat that we get in a ton of the second coal. We get as much heat for $2.15 when we buy the second coal as we get for $3.18 when we buy the first coal.

Example:

Which is cheaper, Pocahontas coal, 15,094 B. t. u. per pound, at $2.90 per ton, or Pennsylvania semi-bituminous coal, 13,155 B. t. u. per pound, at $2.65 per ton? — Answer, Pocahontas is cheaper.

Cost of Power.

We give here some practical examples in finding cost of power, which do not require the evaporative test.

Q. A plant generates electricity at 220 volts, 700 amperes. How many watts and how many kilowatts of power does it give out? — Answer, 700 × 220 = 154,000 watts = 154 K. W.

Q. This plant in a 30-day run consumes 277.2 tons of coal (2,000 pounds to the ton). How many pounds of coal does it consume in 30 days? — Answer, 554,400 pounds.

Q. How many pounds in one day? — Answer, 18,480 pounds.

Q. How many pounds in one hour? — Answer, 770 pounds per hour.

Q. How many pounds per kilowatt hour? — Answer, since the plant generates 154 K. W., it will give out 154 kilowatt hours in one hour. Dividing 770 by 154, we get 5 pounds of coal per K. W. hour.

Q. What is the cost of the coal burned in 30 days at $3 per ton? — Answer, $831.60.

Q. 9,000 cubic feet of water are added to the system in the 30 days. How many gallons of water does this equal? (1 cubic foot equals 7.5 gallons.) — Answer, 67,500 gallons.

Q. What was the cost of the water at 12 cents per 1,000 gallons? — Answer, $8.10.

Q. Cost of oil, waste and grease was $42; labor account, $540; interest, depreciation and taxes, $210;

These figures give only approximate values because of variation in quality, particularly in the case of wood and coal.

	British Thermal Units.
Wood	6,000 per 1 lb.
Bituminous coal	13,000 per 1 lb.
Anthracite coal	12,500 per 1 lb.
Fuel oil	140,000 per gal.
Gasoline and naphtha	125,000 per gal.
Natural gas	1,000 per cu. ft.
Carbureted water gas	600 per cu. ft.
Coal gas	625 per cu. ft.
Water gas	300 per cu. ft.
Raw bituminous producer gas (hot)	250 per cu. ft.
Anthracite producer gas (cold)	145 per cu. ft.

TABLE No. VII.

HEATING VALUE OF DIFFERENT KINDS OF FUEL.

repairs, $53.40. What was the total cost for the 30-day run? — Answer, $1,685.10.

Q. Since there were 154 K. W., how many kilowatt hours were produced in 10 hours? — Answer, 1,540 K. W. H.

Q. How many K. W. H. in 1 day of 24 hours? — Answer, 3,696 K. W. H.

Q. How many K. W. H. in 30 days? — Answer, 110,880 K. W. H.

Q. Since 110,880 K. W. H. cost $1,685.10, what was the cost per kilowatt hour? — Answer, 1.52 cents per K. W. H.

Boiler Horse-power.

The rule commonly used for finding boiler horse-power is this:

A boiler horse-power is the evaporation of 34.5 pounds of water at 212° F. into steam at 212° F.

Examples:

1. We have a 100 horse-power engine which consumes 40 pounds of steam per horse-power hour. What horse-power of boiler will be required?

Solution:

The engine consumes 100 times 40, or 4,000 pounds of steam per hour. Dividing 4,000 by 34.5, we get 115 boiler horse-power.

2. We have a single-valve engine requiring 22 pounds of steam per horse-power hour. This engine is to carry a load of 100 horse-power. What size of boiler should be installed? — Answer, not less than 64 boiler horse-power.

3. We have a four-valve engine of 75 horse-power requiring 16 pounds of steam per horse-power per hour. What size boiler is required? — Answer, not less than 35 boiler horse-power.

4. A condensing engine of 75 horse-power requires 12 pounds of steam per horse-power hour. What size boiler is needed?—Answer, not less than 27 boiler h. p.

ALGEBRA.

Use of Letters to Represent Numbers.

It is often convenient to use **letters** to represent numbers. Thus, if I wished to give a rule for finding the number of square feet in the floor of a room, I should say multiply the length by the width of the room. I can write it in this way.

Length $\times$ width = area.

Now, it is not necessary to write the words length, width and area. Just take the first letter of each word and let that stand for the word. Then we have

$$l \times w = a, \text{ or } lw = a,$$

since in algebra the sign of multiplication need not be written.

That is all an algebraic expression is ; simply letters standing for the names of certain quantities.

Algebra, then, is only a **simplified arithmetic.**

But it is something the engineer and electrician can not get along without, because he can not go far in his calculations unless he learns to take short cuts, and algebra furnishes the short cuts.

We shall give a few more illustrations to show what an algebraic expression is.

Suppose a clothing merchant is invoicing. He finds that he has in a certain stockroom 100 coats, 200 overcoats and 150 hats. He goes through his entire stock in this way, writing down the name of each kind of article. Now, he could save much time if he would let c stand for coats and write 100 c; o stand for overcoats, and write 200 o; h stand for hats, and write 150 h, then this part of his stock would be 100 c, 200 o and 150 h. He might choose other letters to stand for these articles. It doesn't matter what letters are used, provided one letter always stands for the same thing. He could not keep his accounts straight if he let c

stand for coats in one stock and overcoats in another. When he takes his invoice in this way he is using algebra. If he writes the stock in the first room 100 c + 200 o + 150 h he has written an algebraic expression.

By the use of algebra we can express any rule more briefly than we can with words and numbers. Take, for example, the rule for finding the circumference of a circle. To find the circumference, multiply the diameter by 3.1416. So we have

circumference = 3.1416 times diameter;

or, since the diameter equals twice the radius,

circumference = 2 times 3.1416 times radius.

c = 2 times 3.1416 times r.

The number 3.1416 is represented by the Greek letter π (pronounced like "pie"). So we have

$$c = 2 \pi r.$$

We may also write a simple equation for the area of a circle. We have already seen that the area equals the radius multiplied by itself, and this result multiplied by 3.1416,

area = 3.1416 times radius times radius.

By algebra, $a = \pi r^2$.

To find the volume of a square-cornered box or tank, we may let l, w and h stand for length, width and height. Let v stand for volume or contents.

Then v = lwh.

The above examples have been given to show why algebra is used. Many more might be given to show that algebra, when rightly used, makes things much simpler. In each of the examples given we have an equation, one quantity equal to another quantity.

Examples:

1. Using the last equation, find the volume if l = 12 feet, w = 6 feet, h = 4 feet.—Answer, 288 cubic feet.

2. Using the same equation, find the contents of a tank when $l = 15$, $h = 8$, $w = 10$.— Answer, 1,200 cubic feet.

3. From the same equation find what the height must be if $l = 7$ feet, $w = 6$ feet and $v = 168$.— Answer, $h = 4$ feet.

4. In the equation $a = \pi r^2$, if $r = 2$ feet, what does a equal? — Answer, $a = 12.5664$ square feet.

5. In the equation $c = 2\pi r$, if $r = 2$, what does c equal? — Answer, $c = 12.5664$ feet.

We have learned that we can find the volume of a circular tank by multiplying the area of the base by the height.

Now, the equation for the area is as we have just seen:

$$a = \pi r^2.$$

Therefore, if v is the volume and h the height, $v =$ a times h, or ah, which equals $\pi r^2 h$.

Since $a = \pi r^2$, multiplying a by h is the same as multiplying πr^2 by h. So we say $ah = \pi r^2 h$.

Hence $v = ah = \pi r^2 h$.

6. Using the above equation, find the capacity of a circular tank if $r = 3$ feet and $h = 5$ feet.— Answer, 141.372 cubic feet.

7. If a circular tank is to hold 300 cubic feet, and its radius is 4 feet, what must be its height? Use the same equation as in Example 6 and find what number you must multiply πr^2 by to give 300.— Answer, nearly 6 feet.

Adding and Subtracting Algebraic Quantities.

Addition and subtraction in algebra are like addition and subtraction in arithmetic. Addition is like balancing an account.

If a man goes into business with a capital of $10,000, and owes $4,000, his net resources are $6,000.

We may call the capital +10,000 and the debt —4,000. When we add the debt to the capital we get the net resources. The minus sign is used to represent the liability side of the account.

A good example of negative numbers is a below-zero temperature. See Fig. 10. Suppose on a winter day the thermometer reads 10 degrees below zero. The temperature is minus 10, also written —10. This is a minus quantity or a negative quantity. Now, we can add two negative quantities just as we can add two positive quantities. If we add —10 and —5 the result is —15.

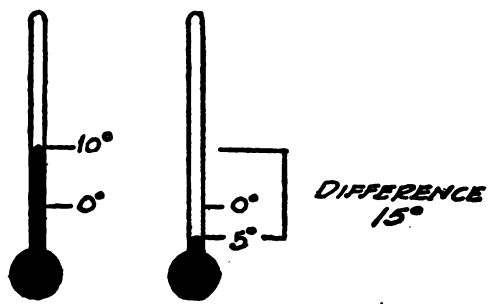


FIG. 10.

If the highest temperature were +10 degrees and the lowest temperature —10 degrees, then the average of these two temperature would be zero; that is, the point half way between the highest and the lowest temperatures would be zero. This illustrates the fact that when we add a plus quantity to an equal minus quantity the sum is zero. We shall see the importance of this when we come to alternating currents.

We can subtract a negative quantity from a positive quantity. Suppose that on a certain day the thermometer reads 10 degrees above zero and at night it

reads 5 degrees below zero. What is the difference between the two temperatures? We must subtract —5 from +10. Now, from the figure we can see that the mercury has fallen 15 degrees. We know then that —5 subtracted from +10 equals +15, or, to state it another way, we know that the difference between +10 and —5 is 15. Now, +5 added to +10 gives 15. We see, then, that to subtract —5 is the same as to add +5.

Examples:

1. Add —10 and —4.— Answer —14.
2. Add —12 and —4.
3. Subtract —6 from +10.— Answer +16.
4. If the highest temperature on a certain day is +20 degrees, and the lowest temperature is —10 degrees, what is the difference between the two temperatures? — Answer, 30 degrees.
5. Add +2 a, —3 a, +4 a.—Answer, +3 a.
6. Add 3 a, 6 b, 7 a, 10 b.— Answer, only quantities of the same kind can be added, so we have 10 a + 16 b.
7. Collect and add the terms of the same kind in the following expression:

6 a + 2 b + 7 c + a + 3 b + 5 c — 4 a.—Answer, 3 a + 5 b + 12 c.

Multiplying and Dividing Algebraic Quantities.

If a merchant has 100 coats, which he represents by 100 c, and doubles his stock, he will then have 200 c. If he divides his original stock into two equal parts he will have in each part 50 c. This illustrates multiplication and division when the signs are all +.

Suppose a man owes $5,000, which we shall represent by —5,000, and to enlarge his business makes his indebtedness three times as great as at first. Then

his indebtedness is represented by —15,000. If he reduces his indebtedness, making it one-fifth what it was at first, we have to divide —5,000 by 5 and we get —1,000. This illustrates division when the dividend is a minus quantity.

In both these illustrations the divisor and the multiplier are + quantities, and this is true in all the practical problems which the engineer and electrician will meet.

In many problems it is necessary to multiply a symbol by a symbol. We had an illustration of this when we multiplied the radius of a circle by itself in finding the area. Now, r is a symbol for the radius of a circle. It stands for the radius of any circle. It may be 1 inch or 10 feet, or any other number. So we can not give r any definite value until we are working with some particular circle. But we can say r multiplied by r, meaning the radius of any circle multiplied by itself. We write it r^2. If we were to multiply 3 r's together, we should write it r^3. The small figure above the r tells how many r's are multiplied together. This number is called the exponent.

If we have a cubical tank, length, width and height equal, and we wish to find the capacity of the tank, we multiply length, width and height together, and, since these three are equal, we are multiplying the same number by itself 3 times. Let l stand for the length of each edge of the tank. Then the capacity is l^3. If we know the capacity and the length of one edge, we can find the area of the bottom by dividing l^3 by l. This gives l^2. Thus, dividing by l makes the exponent 1 less. Multiplying by l makes the exponent 1 greater.

If we have a tank with a square base, but the height different from the length of the base, we can let b represent the length and width of the base and h the height. Then b^2 is the area of the base, and $b^2 h$ is the capacity. Here we are multiplying one symbol by a

different symbol, and we can not actually multiply until we consider some particular tank, so that b and h have definite values. For example, if b = 10 and h = 8, b^2, the area of base, is 100, and $b^2 h$, the capacity, is 800.

Division of one algebraic quantity by another may be expressed in the form of a fraction. Thus, $\frac{a}{b}$ means a divided by b, just as $\frac{2}{3}$ means 2 divided by 3. For example, if a represents the area of a rectangular figure like Fig. 11, and b represents one side, then $\frac{a}{b}$ equals the other side. This statement is strictly true for every rectangular figure, but we can not actually

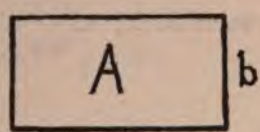


FIG. 11.

divide a by b until we apply it to some particular figure and so have definite numbers for the values of a and b. Thus, if a = 10 and b = 2, the other side of the rectangle $= \frac{10}{2} = 5$.

Fractions in algebra are dealt with in exactly the same way as fractions in arithmetic.

A fraction is multiplied by multiplying its numerator. Thus, 2 times the fraction $\frac{a}{b}$ equals $\frac{2a}{b}$

A fraction is divided by dividing its numerator or

by multiplying its denominator. For example, the fraction $\frac{a}{b}$ divided by 2 equals $\frac{a}{2b}$; the fraction $\frac{6a}{b}$ divided by 2 equals $\frac{3a}{b}$; the fraction $\frac{a^2}{c}$ divided by a equals $\frac{a^2}{c}$.

A fraction is not changed by multiplying both numerator and denominator by the same number. For example, $\frac{a}{b}$ is equal to $\frac{2a}{2b}$ just as ⅔ is equal to ⅙. Multiplying both terms of the fraction by 2 does not change the value of the fraction.

A fraction is not changed by dividing both numerator and denominator by the same number. Thus, ⅚₀ has the same value as ⅓; so $\frac{3a}{6b}$ has the same value as $\frac{a}{2b}$.

It must not be supposed, however, that we can add the same number to both terms of a fraction without changing its value. For example, add 2 to both terms of the fraction ⅛ and we have ⅖, but ⅛ is not equal to ⅖. Neither can we subtract a number from both terms of a fraction. We can not do this either in arithmetic or algebra.

Examples:

1. Multiply 50 a by 3.— Answer, 150 a.
2. Multiply —100 b by 4.— Answer, —400 b.
3. Divide 50 a by 5.— Answer, 10 a.
4. Divide —100 b by 4.— Answer, —25 b.
5. Multiply c^2 by c.— Answer, c^3.

6. Divide t^3 by t.— Answer, t^3.

7. Divide a by c.— Answer, $\frac{a}{c}$

8. Multiply $\frac{c}{d}$ by 2.— Answer, $\frac{2c}{d}$

9. Divide $\frac{c}{d}$ by 2.— Answer, $\frac{c}{2d}$

10. Multiply $\frac{c}{d}$ by c.— Answer, $\frac{c^2}{d}$

11. Divide $\frac{c^3}{d}$ by c.— Answer, $\frac{c^2}{d}$

12. Is the fraction $\frac{c}{d}$ multiplied by 2 if we multiply both terms by 2 and thus get $\frac{2c}{2d}$? — Answer, no, the fraction is not changed in value.

13. Is the fraction $\frac{c}{d}$ changed in value if we add 2 to both terms, thus getting $\frac{c+2}{d+2}$? — Answer, yes, this does change the value of the fraction.

Solving Simple Equations.

The purpose of algebraic equations in engineering is to furnish rules or methods for making useful calculations.

For example, the area of a circle is the square of the radius multiplied by 3.1416. This is expressed very simply by letting a stand for area, r for radius, and the Greek letter π for the number 3.1416. Then we have

$$a = \pi r^2$$

This is an algebraic equation.

The circumference of a circle is 2 times the radius times π:

$$c = 2\pi r$$

The capacity of a circular tank is the area of base times height. Letting v stand for volume or capacity, we have

$$v = \pi r^2 h$$

The capacity of a cubical box or bin is the cube of the length, that is

$$v = l^3$$

It is clear from these illustrations that algebraic equations are time-savers. We shall now discuss methods of solving such equations. In most practical work all the quantities in the equation are known except one, and the object is to find the value of the one unknown quantity. For example, we may know the height and radius of a circular tank, and wish to find the capacity. The equation for capacity of a circular tank gives us the rule for solving the problem. We put the numbers in the equation in place of the letters and then do the work as the equation indicates. If the radius of our tank is 5 feet and the height 8 feet, then for this particular tank

$$v = 3.1416 \times 5^2 \times 8$$

It is necessary to know certain rules that must always be observed in solving equations.

First rule: Whatever is done to one side of an equation must be done to the other side.

If this rule is violated, the equation is no longer true. This rule can be illustrated best with numbers. Take the equation

$$2 + 3 = 5$$

We can add the same number to both sides of the equation. Let us add 4, then we have

$$2 + 3 + 4 = 5 + 4,$$

which is true. But if we add 4 to one side only, we get

$$2 + 3 + 4,$$

which does not equal 5.

Subtracting 3 from both sides, we get

$$2 = 2$$

Multiplying both sides by 2, we get

$$4 + 6 = 10$$

Take the equation

$$a = b$$

Multiply both sides by c, then

$$ac = bc$$

Divide both sides by c, then

$$\frac{a}{c} = \frac{b}{c}$$

Square both sides, then

$$a^2 = b^2$$

Transposing is subtracting the same number from both sides of an equation.

Take the equation

$$a + b = c$$

Subtracting b from both sides, we have

$$a = c - b$$

So when b appears on the right side of the equation it has the minus sign.

Illustrating this with numbers, let

$$a = 2, b = 3, c = 5$$

$$2 + 3 = 5$$

Transposing 3, we have

$$2 = 5 - 3$$

When the 3 appears on the right side, it has the minus sign. This gives us our **second rule, viz.:**

Any term in an equation may be transposed by changing its sign.

Clearing an Equation of Fractions.

Third rule: To clear an equation of fractions, multiply both sides of the equation by a number that will contain the denominators of all the fractions.

For example, take the equation

$$\frac{a}{2} = \frac{b}{3}$$

Multiply both sides by 6, then we have

$$\frac{6\,a}{2} = \frac{6\,b}{3}$$

Or, $3\,a = 2\,b$

Take the equation

$$\frac{a}{b} = \frac{c}{d}$$

The quantity $b\,d$ contains both denominators. Multiplying both sides by $b\,d$, we have

$$\frac{a\,b\,d}{b} = \frac{c\,b\,d}{d}$$

Or, $a\,d = c\,b$,

since $a\,b\,d$ divided by b equals $a\,d$, and $c\,b\,d$ divided by d equals $c\,b$.

We shall now apply these rules in solving some simple equations:

If $2\,x = 10$, what does x equal? Dividing both sides by 2, we get $x = 5$.

If $a + 3 = 10$, what does a equal? Subtracting 3 from both sides, we get $a = 7$.

If $b = 3 \times 2^2$, what is the value of b? $2^2 = 4$, and 3 times $4 = 12$; $b = 12$.

Examples to be solved:

1. $a + 5 = 15$, what does a equal?

2. $\frac{a}{3} = 12$, what does a equal? — Answer, $a = 36$.

3. $3b = 21$, what does b equal?

4. $c = 2\pi r$. If $r = 3$, $\pi = 3.1416$, what does c equal? — Answer, $c = 18.8496$.

5. $v = l^3$. If $l = 8$, what does v equal? — Answer, $v = 512$.

6. $a = \pi r^2$. If $r = 3$, what does a equal? — Answer, $a = 28.2744$.

7. $v = \pi r^2 h$. If $r = 4$, $h = 3$, what is the value of v? — Answer, $v = 150.7968$.

8. For total pressure on piston, we have the equation $P = \pi r^2 p$, in which p is pressure per square inch. If $r = 6$, $p = 50$, what does P equal? — Answer, $P = 5{,}654.88$.

9. In the equation $v = \pi r^2 h$, if $v = 251.328$, $r = 4$, what does h equal? — Answer, $h = 5$.

Hint: $h = \frac{v}{\pi r^2} = \frac{251.328}{\pi r^2}$

Lever, Safety Valve, Torque.

Rules for lever problems can be understood best by considering first one of the simplest levers, the crowbar. The same rules can be applied to any other lever as to the crowbar.

Suppose a man weighing 150 pounds is lifting a weight with a crow bar (Fig. 12). The support on which the bar rests is called the fulcrum. Suppose the man is pushing down on the end of the bar 4 feet from the fulcrum, and the weight is 1 foot from the fulcrum. Then the man's weight will balance a weight of 600 pounds. **The rule is simple. The weight of the**

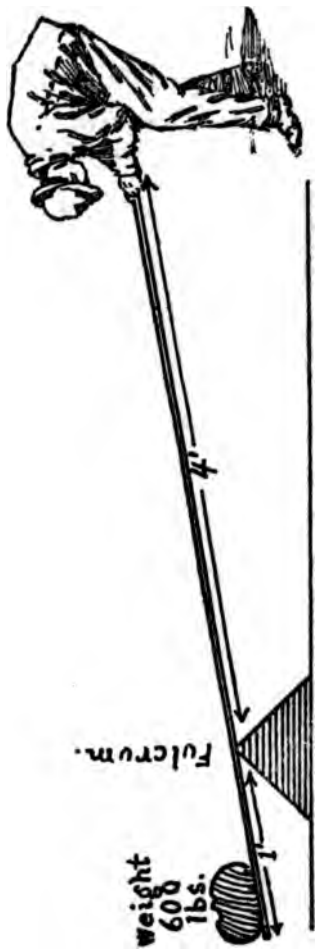


FIG. 12.

man, multiplied by his distance from the fulcrum, equals the weight at the other end times its distance from the fulcrum:

$$150 \times 4 = 600 \times 1$$

The product of the man's weight by his distance from the fulcrum is his turning force. This product is what we call the **torque.** Torque means turning force. The product of weight in pounds times distance from fulcrum in feet is the torque in pound-feet. The torque of the man in this example is 600 pound-feet. The lever rule is that **the total torque which tends to turn the lever in one direction equals the total torque which tends to turn it in the opposite direction.**

In the equation above we have left out of account the weight of the bar itself. Suppose the bar weighs 10 pounds. We must find where this weight is acting. To do this, lay the bar on the fulcrum, with no weight on it, and move it along until it balances. Now, the point where it balances is where the weight of the bar acts. This is called the center of gravity of the bar. Let us suppose it balances at the middle. The middle point of the bar is 1½ feet from the fulcrum. This makes the turning force or torque of the bar itself 10 times 1½, or 15 pound-feet. This added to the turning force of the man makes the total turning force on the man's side of the fulcrum 615 pound-feet. The turning force on the other side must also be 615 pound-feet. But this turning force equals the weight times 1 foot. Hence the weight is 615 pounds.

Fig. 13 shows a lever safety valve. The force which is trying to raise the valve is the pressure of the steam. The forces which are holding down the valve are the weight of the ball, the weight of the lever, and weight of valve and spindle. Now apply the lever rule. The turning force of the steam at blowing-off pressure equals the combined turning

forces of the ball, lever, valve and spindle. The total pressure of steam equals pressure per square inch times area of valve. Turning force of ball equals weight of ball times distance from ball to fulcrum.

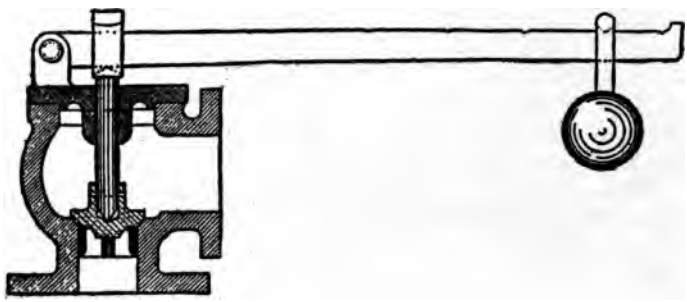


FIG. 13.

THE LEVER SAFETY VALVE.

Turning force of lever equals weight of lever times distance from fulcrum to center of gravity of lever. Turning force of valve and spindle equals weight of valve and spindle times distance from fulcrum to center of valve.

Let W = weight of ball in pounds.
v = weight of valve and spindle in pounds.
w = weight of lever in pounds.
L = distance between fulcrum and center of ball in inches.
l = distance between fulcrum and center of valve in inches.
g = distance between fulcrum and center of gravity of lever in inches.
A = area of valve in square inches.
P = pressure of steam, in pounds per square inch, at which valve opens.

Then, turning force of steam equals

$$P\,A\,l$$

and turning force of ball, lever and valve, taken together, equal

$$W\,L + v\,l + w\,g.$$

Therefore:

$$P\,A\,l = W\,L + v\,l + w\,g.$$

In practice, g may be taken as one-half the length of the lever. The fulcrum is the bolt which passes through the end of the lever.

To find steam pressure at which valve will open, divide both sides of the above equation by A l. This gives

$$P = \frac{W\,L + v\,l + w\,g}{A\,l}$$

which means that blowing-off pressure equals combined turning forces of lever, valve and ball, divided by area of valve times distance from fulcrum to center of valve.

Example:

Weight of ball W = 76 pounds.

Weight of valve and spindle v = 4 pounds.

Weight of lever w = 5.5 pounds.

Distance between fulcrum and center of ball L = 20.15 inches.

Distance between fulcrum and center of valve l = 2 inches.

Distance between fulcrum and center of gravity of lever = ½ length of lever g = 17 inches.

Area of valve A = 12.56 square inches.

Find the blowing-off pressure.

Solution:

$$W\,L = 1{,}531.4$$
$$v\,l = 8$$
$$w\,g = 93.5$$
$$A\,l = 25.12$$
$$W\,L + v\,l + w\,g = 1{,}632.9$$
$$\frac{1{,}632.9}{25.12} = P = 65 \text{ pounds}$$

When gauge reads 65, valve will open.

To find where the ball must be placed so that valve will open at a certain pressure:

L is the quantity to be found.

In the equation

$$P\,A\,l = W\,L + v\,l + w\,g,$$

transpose v l and w g. This gives

$$P\,A\,l - v\,l - w\,g = W\,L.$$

Divide by W. This gives

$$\frac{P\,A\,l - v\,l - w\,g}{W} = L.$$

Example:

With the same safety valve as in the last example, find where the ball must be placed so that the valve will open at 60 pounds pressure.

Solution:

$$P\,A\,l = 1{,}507.2$$
$$v\,l = 8$$
$$w\,g = 93.5$$
$$W = 76$$
$$P\,A\,l - v\,l - w\,g = 1{,}405.7$$
$$\frac{1{,}405.7}{76} = 18.4$$

Center of ball must be 18.4 inches from the fulcrum.

To find what weight of ball should be used when

in all three parts taken together, we have 225 square inches. So we have used the total number of square inches that we had to begin with, and we have made

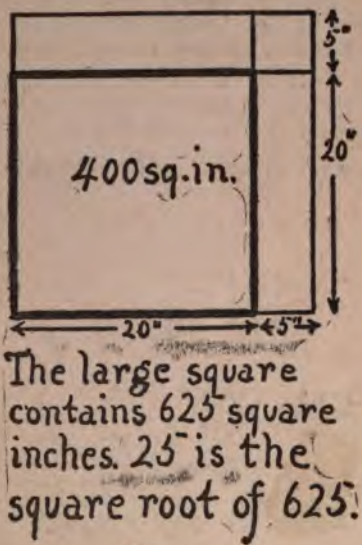


FIG. 14.

a square, each side of which measures 25 inches. Therefore, the square root of 625 is 25. This gives us the

Rule for Finding Square Root.

Divide the number into periods of two figures each, beginning with units. Find the largest square in the left-hand period. Subtract this square from the left-hand period and place its square root in the answer. Bring down and annex the next period; call this the remainder. Take twice the part of the answer already found as a trial divisor. Find how many times the

trial divisor is contained in the remainder and place this figure as the second figure in the answer; also add this number to the trial divisor, to make the complete divisor. Multiply the complete divisor by the last figure in the answer and subtract this result from the remainder. Bring down the next period and repeat the operation. Continue in the same way until there is no remainder, or until the root has been found to as many decimal places as desired.

The arithmetical work is as follows:

$$\begin{array}{r|l} & \overline{6\,25}(25 \\ & 4 \\ \hline 40 & 225 \\ 5 & 225 \\ \hline 45 & \end{array}$$

Find the square root of 6,724.

$$\begin{array}{r|l} & \overline{67\,24}(82 \\ & 64 \\ \hline 160 & 324 \\ 2 & 324 \\ \hline 162 & \end{array}$$

Find the square root of 54,756.

$$\begin{array}{r|l} & \overline{54\,75\,6}(234 \\ & 4 \\ \hline 40 & 147 \\ 3 & 129 \\ \hline 43 & \\ \hline 460 & 1856 \\ 4 & 1856 \\ \hline 464 & \end{array}$$

To find the square root of a number which contains a decimal, proceed in the same way, taking care to begin at the decimal point to divide the number into periods.

For example: The square root of 6.25 is 2.5. But suppose we are to find the square root of 62.5. The work is as follows:

$$\begin{array}{r|l} & \widehat{62.5}\widehat{0}(7.9+ \\ & 49 \\ \hline 140 & 1350 \\ 9 & 1341 \\ \hline 149 & \end{array}$$

The square root is 7.9+.

Examples:

1. Find the square root of 576.— Answer, 24.

2. The area of a square floor is 256 square feet. What is the length of one side? — Answer, 16 feet.

3. I wish to make a bin to hold 100 tons of coal. The coal requires 44.1 cubic feet to the ton. If the bin is 10 feet high and the bottom square, what is the length of one side? — Answer, 21 feet.

4. What is the radius of a piston if its area is 201.0624 square inches? Divide by 3.1416 and find the square root of the quotient.— Answer, 8 inches.

5. What is the radius of a piston if its area is 226.9806 square inches? — Answer, 8.5 inches.

6. A safety valve is to be designed for a total pressure of 600 pounds, 50 pounds per square inch. What must be the radius of the valve? Divide 600 by 50 to find area, and this result by π to find r^2. Then find the square root.— Answer, 1.9 inches.

Cube Root.

The cube root of a number is the quantity which, multiplied by itself three times, will produce the given number. The cube root may be regarded as the length of the edge of a cube, the volume of the cube being the given number.

Square root and cube root of numbers may be found from Table IX.

Heating and Ventilation.

We shall explain here heating and ventilating calculations, as affecting the work of the operating engineer.

Following are some facts which the engineer should know regarding his heating plant:

One horse-power is equivalent to 42.7 heat units per minute.

One horse-power represents the energy required to evaporate .044 pound of water per minute.

With steam heat, 1 horse-power is sufficient to supply 100 square feet of direct cast-iron radiation, or 90 square feet of pipe or heating-coil radiation.

The heat that will raise 1 pound of water 1 degree will raise 4½ pounds of air, or 9 pounds of iron, 1 degree.

At ordinary pressure and temperature, about 12 cubic feet of air weigh 1 pound.

One square foot of grate surface supplies 36 square feet of boiler surface, and this carries 196 square feet of dried radiating surface for heating. Area of chimney should allow 49 square inches for 1 square foot of grate surface.

For low-pressure steam heating, the size of the chimney may be somewhat less.

A cast-iron radiator will radiate much less heat when enameled than when painted with bronze or a mineral paint.

In estimating radiation, a square foot of glass and a square yard of ordinary outside wall have about the same cooling value.

Mills' rule for computing amount of radiation: "To find the amount of radiation required to heat a room with low-pressure steam to 70° F. when the outside temperature is at 0° F., allow 1 square foot of radiation for each 200 cubic feet of contents, 1 square foot of radiation for each 20 square feet of outside wall surface, and 1 square foot of radiation for each 2 square feet of glass surface (counting outside doors as glass surface). The sum of these results will be the amount of radiation required. For hot water, add 60 per cent to this result."

Amount of carbon dioxid usually present in air is 2 to 4 parts in 10,000, and this amount is not harmful.

Effect of Different Modes of Lighting on the Air, According to Doctor Tidy.

Material sufficient to produce light equal to 12 standard candles, 1 hour.	Cubic feet of oxygen consumed.	Cubic feet of carbon dioxid given off.	Cubic feet of air vitiated.
Illuminating gas	5.45	3.21	345.25
Sperm oil	4.75	3.33	356.75
Sperm candles	7.51	5.77	614.85
Wax candles	8.41	5.90	632.25
Electric light	None	None	None

The value of electric light in keeping the air pure is apparent from this table.

The least amount of air supply essential to good ventilation is given in the following table prepared by Doctor Billings:

	Cubic feet per hour.
Shops	3,600 per person.
Hospitals	3,600 per bed.
Schools and churches	2,400 per person.
Theaters and ordinary halls of audience	2,000 per seat.
Offices and dining-rooms	1,800 per person.

To find what amount of air is being supplied, use an anemometer to measure the velocity of the air in the supply pipe.

The anemometer gives velocity in feet per minute.

Velocity of air in feet per minute, multiplied by area of air pipe in square feet, gives cubic feet of air per minute. This result multiplied by 60 gives cubic feet per hour.

Another method is to find the pressure of the air by means of a water-pressure gauge. Then its velocity can be found from Table 13.

The amount of moisture in the air is important. For health the air, whatever its temperature, should contain about 70 per cent as much moisture as it can hold at that temperature.

Warm air can hold more moisture than cold air.

The effect of heating air on its humidity is shown in the following table. This table supposes the outside air to be saturated, and gives the humidity when brought in and heated to 70° F.

Temperature of outside air, humidity 100 per cent.	Humidity of the same air when heated to 70° F.
0	8.5 per cent.
10	12.3 per cent.
20	17.9 per cent.
30	25.5 per cent.
40	36.5 per cent.
50	51.6 per cent.
60	72.0 per cent.

Per cent of humidity is found by means of a wet and dry bulb hygrometer. This consists of two thermometers, one having the bulb wrapped with a wick which dips in water The dryer the air the more rapidly water evaporates from the wick and the lower the

reading of the wet-bulb thermometer. The instrument is furnished with scales, giving per cent of humidity for different readings of the thermometers.

The Gas Engine.

To Find the Indicated Horse-power.

The indicated horse-power of a gas engine is found in the same way as that of a steam engine, except that the pressure is acting on the piston only in one stroke out of four if a four-cycle engine is used. Since the fly-wheel makes one revolution for two strokes of the piston, the number of working strokes is one-half the number of revolutions, provided there are no missed explosions.

The mean effective pressure is found from the indicator-card, as explained for steam engines. Then if P is the mean effective pressure, A the area of piston, N the number of working strokes per minute and L the length of stroke in feet, I. H. P. indicated horse-power,

$$\text{I. H. P.} = \frac{\text{P L A N}}{33{,}000}$$

Example:

1. Find indicated horse-power if mean effective pressure is 54 pounds per square inch, revolutions per minute 356, area of piston 18.25 square inches, length of stroke ¾ foot.

Solution:

$$\text{Working strokes per minute} = \frac{356}{2} = 178$$

$$54 \times 178 \times 18.25 \times \tfrac{3}{4} = 131{,}564.25$$

Answer, 3.9 H. P.

2. Find the indicated horse-power if the mean effective pressure is 75 pounds per square inch, r. p. m. 600, area of piston 18.75 square inches, length of stroke ½ foot.— Answer, 6.3 H. P.

To Find the Brake-horse-power.

The brake-horse-power of a gas or gasoline engine is found in the same way as the brake-horse-power of an electric motor. The brake-horse-power is the power given out by the engine. It is the power that is available for running machinery.

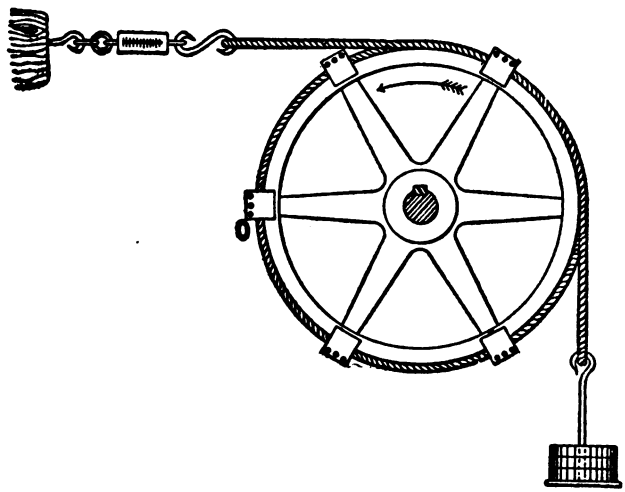


FIG. 15.

BRAKE HORSE-POWER.

The calculations for a brake test are as follows:

Let F = pull in pounds at rim of pulley, found by spring balance or weight method. (Fig. 15.)

d = diameter of pulley in feet.

n = number of revolutions per minute.

B. H. P. = brake-horse-power.

Then

πd is the circumference of pulley.

$\pi d n$ is the distance rim of pulley travels in 1 minute.

$\pi d n F$ is the number of foot-pounds of work in 1 minute.

And B. H. P. $= \frac{\pi d n F}{33{,}000}$

In making this test the pulley becomes very hot, and an arrangement must be provided for cooling it with water.

Example:

If F = 90 pounds, d = 16 inches = 1⅓ feet, n = 356, what is the B. H. P.? — Answer, 4 H. P.

Mechanical Efficiency of a Gas Engine.

The efficiency of an engine or a motor is the ratio of the power output to the power input, or brake-horse-power divided by indicated horse-power:

$$\text{Mechanical efficiency} = \frac{\text{B. H. P.}}{\text{I. H. P.}}$$

The efficiency is usually expressed in per cent.

Example:

What is the efficiency of a gas engine if the indicated horse-power is 30 and the brake-horse-power is 21.— Answer, .7, or 70 per cent.

Gas Used per Horse-power per Hour.

The gas used is measured by running it through a gas meter. The quantity of gas used is divided by the time in hours, and this result by the horse-power. If we divide by the indicated horse-power, we get the amount of gas used per I. H. P. per hour. If we divide by the brake-horse-power, we get the amount of gas used per B. H. P. per hour.

Heat Efficiency of a Gas Engine.

Part of the heat supplied to a gas engine is wasted. We can find what per cent of the total heat supplied to the engine is utlized in work as follows: Multiply the heating value of a cubic foot of the gas used by the number of cubic feet of gas supplied to the engine in 1 hour. This gives the total number of heat units (B. t. u.) supplied to the engine per hour. Now, 1 heat unit equals 778 foot-pounds of work. Hence, if we multiply the brake-horse-power by 33,000, and this result by 60, we get the number of foot-pounds of work given out by the engine in 1 hour. Dividing this result by 778, we get the number of heat units changed into work in the engine in 1 hour. Dividing this last result by the number of heat units of the gas consumed in 1 hour gives the heat or thermal efficiency.

Example:

What is the thermal or heat efficiency of a gas engine of 10 B. H. P. which consumes 135 cubic feet of gas per hour, the heat value of the gas being 750 B. t. u. per cubic foot?

Solution:

Heat units received $= 135 \times 750 = 101{,}250$ B. t. u.

Equivalent in work $= 101{,}250 \times 778 = 78{,}772{,}500$ foot-pounds per hour.

Work given out in 1 hour $= 33{,}000 \times 60 \times 10 = 19{,}800{,}000$.

$\frac{19{,}800{,}000}{78{,}772{,}500} = .251$.— Answer, 25.1 per cent.

For gasoline engines the same method is followed, except that instead of cubic feet of gas we must find the number of pounds of gasoline consumed in 1 hour and multiply by the heat value of 1 pound of gasoline. The rest of the calculation is the same as for the gas engine.

If a test of the heat values of the gas or gasoline used can not be made, the following values may be taken as approximately correct:

Gasoline.....27,000 to 28,000 B. t. u. per lb.
Ordinary illuminating gas.....590 B. t. u. per cu. ft.
Natural gas .....1,000 B. t. u. per cu. ft.
Water gas (not carbureted)..325-340 B. t. u. per cu. ft.

Refrigeration.

In refrigeration the essential thing is **latent heat.** We have discussed heat of vaporization of water in connection with boilers. Under atmospheric pressure, water boils at 212°. The water and the steam are both at 212°, yet heat must be supplied to the water constantly to keep it boiling. We must supply 965.7 British thermal units of heat for every pound of water evaporated. This heat is said to be "latent," that is, "hidden" in the steam. It is not "sensible" heat, because it does not affect the thermometer. The water in evaporating takes this heat from the fire and renders it latent in the steam. This latent heat is given out again as "sensible" heat when the steam condenses.

There is also latent heat of melting. When ice melts, the ice and the water when first formed are at the same temperature, 32° F. Yet heat must be given to the ice to melt it. It requires 142 B. t. u. to melt 1 pound of ice. This heat becomes latent in the water. The same amount of heat must be taken out of the water at 32° to freeze it.

Now, every liquid has its boiling point and its latent heat. When a liquid vaporizes it takes up the latent heat of evaporation from the surrounding substances.

Ammonia gas, which is commonly used for refrigeration, is liquefied by pressure and then allowed to

expand and vaporize. Its latent heat depends on its temperature and pressure. At a gauge pressure of 15.67, and a temperature of 0° F., the latent heat of ammonia is 555.5 B. t. u. At 23.85 pounds and 10° the latent heat is 549.35. At 44.71 pounds and 30° the latent heat is 536.92. The amount of refrigeration that can be accomplished depends on the latent heat, which in turn depends on the pressure and temperature at which the liquid is vaporized.

Carbon dioxid at 0° F. and 21.2 atmospheric pressure has a latent heat of 124. Sulphur dioxid at 50° F. and 18.55 pounds has a latent heat of 160 B. t. u. Both these substances are used for refrigeration.

Any liquid that boils below ordinary atmospheric temperatures may be used as a refrigerant.

The refrigerant must first be compressed until it is liquefied. Compression heats the substance. The heat of compression must be removed from the condenser. The amount of heat to be removed is figured as equal to the latent heat of vaporization plus a small amount due to friction in the condenser. The heat is removed either by the evaporation of water sprayed or poured over the condenser pipes, or by submerging the condenser pipes in cold water.

About 510 pounds of ammonia expanded from liquid to vapor produce as much refrigeration as can be obtained by melting 1 ton of ice. This amount of ammonia, vaporized at 0° and 15 pounds pressure, occupies 4,650 cubic feet of space. About 25 per cent capacity should be added to this amount to make up for losses. With 15 pounds suction or return pressure, 7,000 cubic inches of ammonia gas must be pumped per minute on the average continuously for twenty-four hours to produce 1 ton of refrigeration. With 20 pounds return pressure, 6,000 cubic inches per minute per ton capacity daily.

Formula for capacity of compressor:

$$C = \frac{P\,L\,N}{T}$$

where C=capacity in tons of refrigeration.
P=area of piston in square inches.
N=number of strokes per minute.
T=cubic inches gas per minute per ton refrigeration daily.

In applying the above formula, a correction must be made for clearance. To make this correction, multiply area of piston in inches by clearance in inches, and this product by the number of strokes per minute. Multiply this product by the quotient obtained by dividing the condenser pressure by the evaporator pressure. The product will represent the loss on account of clearance. Subtracting this result from the displacement first obtained gives the net or actual displacement or capacity.

Example:

Find the capacity of a compressor with 9-inch bore and 12-inch stroke, running at 100 strokes per minute, cylinder having $\frac{1}{16}$-inch clearance condenser pressure, 185 pounds; evaporator pressure, 20 pounds.

Area of piston = $9 \times 9 \times .7854 = 63.62$
P L N = $63.62 \times 12 \times 100 = 76{,}344$
$\frac{1}{16} = .0625$
$.0625 \times 63.62 \times 100 = 397.625$
$185 \div 20 = 9.25$
$397.625 \times 9.25 = 3{,}678$
$76.344 - 3{,}678 = 72.666$
$72.666 \div 6{,}000 = 12.1$ tons daily.

The time required to freeze a cake of ice is proportional to the square of the thickness of the cake. For example, it takes four times as long to freeze 2-inch ice as to freeze 1-inch ice, and one hundred times as long to freeze 10-inch ice as to freeze 1-inch ice.

According to one of the leading refrigerating engineers and builders (E. T. Skinkle) the amount of condenser pipe required is as follows:

Ice-making Capacity, Tons.	Refr. Capacity, Tons.	Size of Pipe.	No. of Feet per Ton Ice.	No. of Feet per Ton Refr.
12	25	1″	294.4	147.2
20	35	1″	222.	126.8
30	50	1″	258.3	155.
40	75	1¼″	193.75	103.33
50	100	1″	279.	139.5
60	125	1¼″	206.6	99.2

Average 1¼-inch pipe per ton ice-making, 192.12.
Average 1¼-inch pipe per ton refrigeration, 98.79.

The difference in pressures and power required with different condenser-water temperatures is shown in the following table:

Temperature Condenser Water Degrees F.	Temp. of Gas in Condenser.	Corresponding Cond. Pr. (Lbs. Gage.)	H. P. per Ton Refr. with Zero in Evap.
55	70°	115	.88
60	75°	127	.95
65	80°	139	1.02
68	88°	153	1.09
70	90°	168	1.16
75	95°	184	1.24
80	100°	200	1.32
90	110°	225	1.75

Horse-power of an Electric Motor.

The brake-horse-power of an electric motor is found in the same way as the brake-horse-power of a gas engine. The rule for this operation has already been given. The brake-horse-power is the horse-power output.

The horse-power input, or electrical horse-power, is found by dividing the number of watts received by 746. The number of watts is found by multiplying volts by amperes.

$$\text{Horse-power input} = \frac{\text{watts}}{746} = \frac{\text{volts} \times \text{amperes}}{746}$$

The commercial efficiency of a motor is the horse-power output divided by the horse-power input and expressed as a per cent.

Examples:

1. What is the electrical horse-power of a motor that takes a current of 25 amperes on a 220-volt circuit? — Answer, $\frac{25 \times 220}{746} = 7.3$ H. P.

2. What is the efficiency of the same motor if its output is 5 horse-power? — Answer, 68.4 per cent.

Trigonometry.

Measuring Angles and Triangles.

The measurement of angles and triangles is a very simple matter. Suppose you have a hinged ruler (Fig. 16) and open it a little, the opening between the two sides of the ruler is an angle. Open it wider and the angle is larger. When the ruler is shut the angle is zero. An angle is the opening between two lines that meet in a point.

Angles are measured in **degrees.** In the circumference of a circle there are 360 degrees. Suppose

we have a single armature coil. If it makes a complete turn it has turned through 360 degrees. If it makes a quarter turn it has turned through 90 degrees, a half turn 180 degrees, and so on.

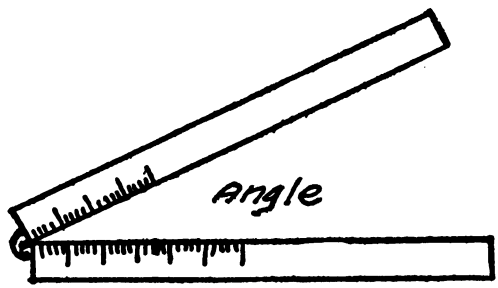


FIG. 16.

The engineer must understand the **measurement** of angles and the sines and cosines of angles in order to understand the **sine curve** which is used in calculations of alternating currents. The important thing, however, is the **power factor**, and this can not be understood without a knowledge of **cosines** of angles. It is true there are tables giving power factors, but unless you understand cosines you can not understand what the power factor means, nor why the **true** watts are only a certain fraction of the **apparent** watts.

In Fig. 17 the **sine, cosine** and **tangent** of the angle are clearly shown. Suppose the line **OC** is the line that is rotating around the point **O**. The point **C** at the end of the line marks out a circle at it goes around. When it has turned through the angle **COB**, the height of **C** above the base line is **CB**, and this is the **sine** of the angle. The distance of **C** from the vertical line **AO** is **CE**, and this is the **cosine** of the angle. It can easily be seen that as **C** turns in the direction of the **arrow** the sine grows **larger** and the cosine **becomes**

smaller. When **OC** has made a quarter turn and reached the position **OA**, the cosine is zero and the sine equals **OA** or the radius of the circle. After **OC** passes **OA**, the sine becomes smaller, and the cosine is to the left of A.

In the above definitions we have assumed that the radius of the circle is 1. For example, if the radius is 1 inch, then the line CB is a certain fraction of an inch, and this fraction is the sine of the angle COB. If the radius is not 1, then we must divide CB by the radius to get the sine of the angle, and for the same angle we shall get the same fraction as before. The clearest idea of the sine, however, can be obtained by assuming the length of the radius to be 1. Then

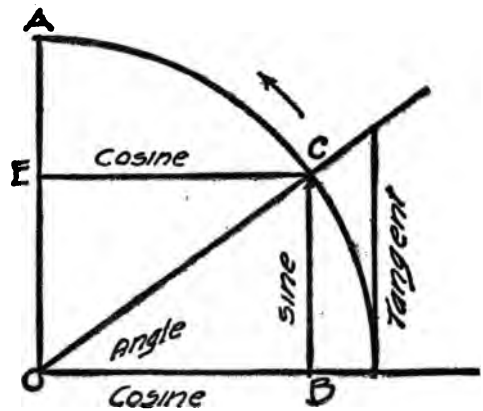


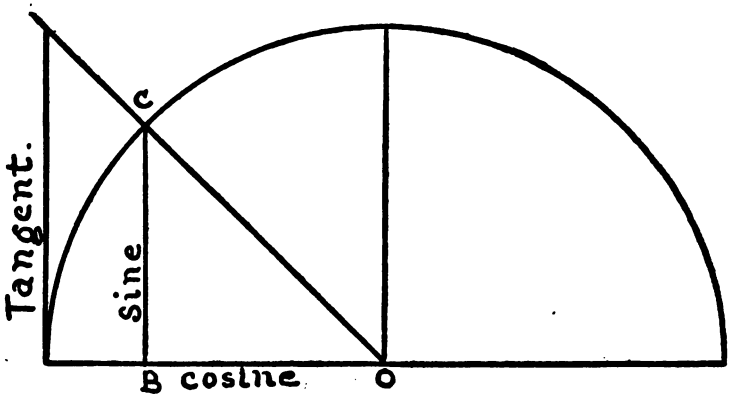
FIG. 17.

the actual length of the line CB is the sine, the actual length of EC is the cosine, and the length of line marked tangent is the tangent of the angle.

It is possible with any angle to draw a circle whose radius is one unit, with the center of the circle at the

point where the lines forming the angle meet. Then the sine, cosine and tangent lines can be drawn as in Fig. 17.

It can readily be seen, by referring to Fig. 17, that the farther the line OC turns in the direction of the arrow the longer the sine and tangent lines become, and the shorter the cosine line becomes. So it is clear that as the angle increases from 0° to 90° the sine and



Sine, cosine, and tangent of an angle greater than 90° and less than 180°

FIG. 18.

tangent become larger and the cosine becomes smaller. At 90° the cosine is 0, the sine is 1, or equal to the radius, and the tangent is so long that we can not measure it, and we call it infinity.

If the angle is greater than 90° and less than 180°, the cosine line, as Fig. 18 shows, is measured to the left from the vertical line AO. We call it a minus

cosine, because it is opposed in direction to the plus cosine of our first angle, which is between 0° and 90°. Between 180° and 270° the sine is minus, because it is measured downward from the horizontal line OB and is opposed in direction to the plus sine of the first angle. Between 270° and 360° the sine is minus and the cosine is plus. Table XIII gives the sines, cosines and tangents of some common angles.

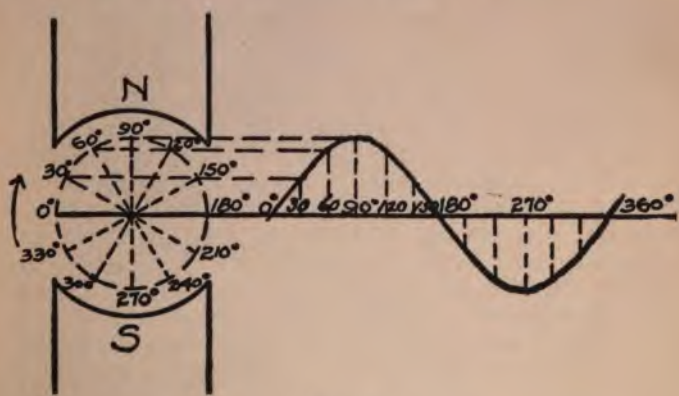


FIG. 19.

Fig. 19 shows the sine curve for a single armature coil. You will observe that the dotted lines marked 30°, 60°, etc., are equal to the sines of these angles. The first dotted line in the curve for example represents the distance the coil is from the horizontal line when it has turned 30 degrees. It also represents the rate at which the wire is cutting magnetic lines of force. It also represents the electrical pressure, and that is the important thing. The pressure is proportional to the sine of the angle.

The wavy line to the right of the circle is therefore

the pressure curve. It corresponds exactly to the steam-pressure curve marked out by the steam-engine indicator. The height of this curve above the horizontal line for any angle from 0° to 180° represents the electrical pressure, or voltage, when the coil is at that particular angle. This height, as we have seen, is the sine of the angle.

At 90° the curve has its greatest height, and the coil is perpendicular to the 0 line. The pressure is greatest at this point. The pressure for any other angle is, therefore, the maximum pressure; that is, the 90° pressure multiplied by the sine of the angle. For example, the sine of 30° is one-half, and the electrical pressure at 30° is one-half of the pressure at 90°.

From 180° to 360° the curve is below the horizontal line, which means that the pressure is minus, or in the reverse direction from the first. The pressure is, therefore, alternating in direction, and we have an alternating current in the armature. If armature is connected to line by collector rings instead of commutator, there will be an alternating current in the line. The distance below the line, that is, the sine of the angle, measures the amount of this minus pressure for any angle.

Fig. 20 is the pressure curve for two armature coils wound at an angle of 90° apart on the same armature. The heavy wave lines are the curves for the two coils separately. The dotted wave line shows the line pressure produced by the two coils acting together. For any angle the height of the dotted line above the horizontal is the sum of the heights of the two heavy lines. At 45° the dotted line crosses the horizontal. At this point one heavy line is as far above the horizontal as the other is below it; that is to say, the pressure in one coil is plus and the pressure in the other coil is equal and in the opposite direction to the first. The second pressure is, therefore, minus. These two pres-

sures acting together actually produce zero pressure in the line. The last statement explains the fact that one pressure is called plus and the other minus. If we add a plus quantity to an equal minus quantity the sum is zero. Fig. 20 is the pressure curve for a

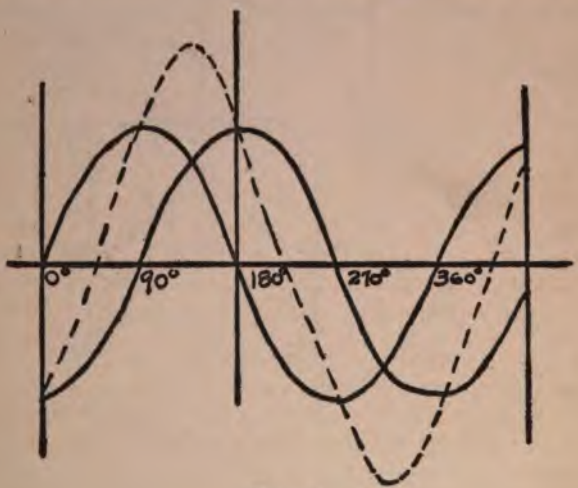


FIG. 20.

two-phase current. When one pressure is at its greatest value the other is at zero, because when one coil is at 90° the other is at 0°. Fig. 21 is the pressure curve for three coils wound on the same armature and 120° apart. This is the curve for a three-phase current.

Idle Current and Power Factor.

In an alternating-current circuit the actual power lags a certain angle behind the apparent power. The curves, Figs. 19 and 20, represent voltage. Voltage

multiplied by the whole current equals apparent power. The actual power is the apparent power multiplied by the cosine of the angle of lag. The cosine of the angle of lag is the power factor. Expressed in percentage,

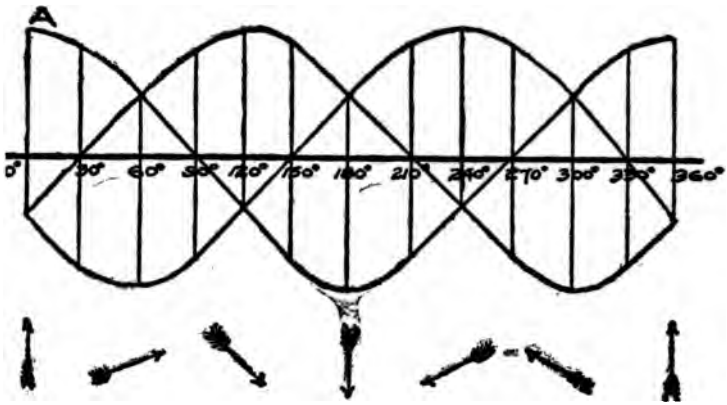


FIG. 21.

the power factor is the cosine of angle of lag multiplied by 100. To find actual power in watts, multiply apparent power in watts by power factor or cosine of angle of lag:

True power = volts × amperes × power factor.

The wattless current, or idle current, is the actual current multiplied by the sine of the angle of lag. Table XIV gives sines, cosines and power factors for angles from 0° to 90°.

Examples:

1. What is the idle current if the angle of lag is 40° and the actual current 30 amperes? — Answer, 19.2 amperes.

2. What is the true power if the apparent power

is 2,000,000 watts, that is, 2,000 kilowatts, and the angle of lag 25°? — Answer, 1,812.6 kilowatts.

3. If the voltage is 20,000, current 500 amperes, angle of lag 20°, what is the true power? — Answer, 9,397 kilowatts.

Angle.	Sine.	Cosine.	Tangent.
0°	0	1	0
30°	$\frac{1}{2} = 0.5000$	$\frac{\sqrt{3}}{2} = 0.8660$	$\frac{1}{\sqrt{3}} = 0.5773$
45°	$\frac{1}{\sqrt{2}} = 0.7071$	$\frac{1}{\sqrt{2}} = 0.7071$	1
60°	$\frac{\sqrt{3}}{2} = 0.8660$	$\frac{1}{2} = 0.5000$	$\sqrt{3} = 1.732$
90°	1	0	∞
120°	$\frac{\sqrt{3}}{2} = 0.8660$	$-\frac{1}{2} = -0.5000$	$-\sqrt{3} = -1.732$
135°	$\frac{1}{\sqrt{2}} = 0.7071$	$-\frac{1}{\sqrt{2}} = -0.7071$	-1
150°	$\frac{1}{2} = 0.5000$	$-\frac{\sqrt{3}}{2} = -0.866$	$-\frac{1}{\sqrt{3}} = -0.5773$
180°	0	-1	0

TABLE NO. X.

The Difference Between a Kilowatt and a Kilowatt Hour.

There is the same difference between a kilowatt and a kilowatt hour that there is between a horse-power and a horse-power hour.

The output of a 1 horse-power engine running 1 hour is 1 horse-power hour.

Q. If a 1 horse-power engine runs 10 hours, how much greater is its output than if it runs 1 hour? — Answer, the output is 10 times as great.

Angle	Cosine of Angle	Power-factor, in Percentages	Sine of Angle	Idle Current, in Percentages
0°	1	100	0	0
5°	0·9962	99·62	0·0872	8·72
10°	0·9848	98·48	0·1737	17·37
15°	0·9659	96·59	0·2588	25·88
20°	0·9397	93·97	0·3420	34·20
25°	0·9063	90·63	0·4226	42·26
30°	0·8660	86·60	0·5000	50·00
35°	0·8190	81·90	0·5736	57·36
40°	0·7660	76·60	0·6428	64·28
45°	0·7071	70·71	0·7071	70·71
50°	0·6428	64·28	0·7660	76·60
55°	0·5736	57·36	0·8190	81·90
60°	0·5000	50·00	0·8660	86·60
65°	0·4226	42·26	0·9063	90·63
70°	0·3420	34·20	0·9397	93·97
75°	0·2588	25·88	0·9659	96·59
80°	0·1737	17·37	0·9848	98·48
85°	0·0872	8·72	0·9962	99·62
90°	0·000	0·00	1·000	100

TABLE No. XI.

LINE, COSINE AND POWER FACTORS.

Q. If a 1 horse-power engine runs 10 hours, how many horse-power hours does it give out? — Answer, 10 horse-power hours.

Q. Is it then a 10 horse-power engine? — Answer, no; it is still a 1 horse-power engine.

Q. What is the difference, then, between horse-

power and horse-power hour? — Answer, horse-power is the capacity of the engine or the work it can do if it runs a specified time. The horse-power of an engine remains the same, no matter how long it runs. One horse-power hour is the output of a 1 horse-power engine running 1 hour. The output of an engine in horse-power hours depends both on its horse-power and on the length of time it runs. Power is measured in horse-power. Output or work done may be measured in horse-power hours.

Q. What is the difference between a kilowatt and a kilowatt hour? — Answer, a kilowatt is 1⅓ horse-power. A kilowatt is a measure of power. A kilowatt hour is the energy delivered or work done by 1 kilowatt running for 1 hour.

Q. If power at the rate 1 kilowatt is delivered for 10 hours, how many kilowatt hours of energy are delivered? — Answer, 10 kilowatt hours.

Q. If 10 kilowatts are delivered for 1 hour, how many kilowatt hours are delivered? — Answer, 10 kilowatt hours; 10 kilowatts for 1 hour will do the same amount of work as 1 kilowatt for 10 hours.

Q. How can a 50 K. W. machine be made to have an output of 500 K. W. H.? — Answer, simply run it 10 hours.

Q. Why do you pay for electricity in kilowatt hours and not in kilowatts? — Answer, because if we paid for a kilowatt of power we could use that power for any length of time. In that case we would pay the same, no matter how long we used the power. One man might get a hundred times as much electrical energy as another man for the same money. If you hire a team to do a job of work you pay for the length of time the team works.

Q. Why do we speak of the capacity of a plant in K. W. and not in K. W. H.? — Answer, simply because

BROWN & SHARPE.	Diameter.		Square of Diameter.	Sectional Area.	Safe Carrying Capacity in Amperes.				Weight.				Length		Resistance of Pure Copper Wire at 75 Degrees F. (23.89 Deg. Cent.)			
							Chicago Code.		Bare		Weather-proof Insulation		Bare.	Weather-proof Insulation.				
No.	Mils.	Millimetres.	Circular Mils.	Square Millimetres	National Code.	New England Code.	Concealed Work.	Open Work.	Pounds per 1,000 Feet.	Pounds per Mile.	Pounds per 1,000 Feet.	Pounds per Mile.	Feet per Pound	Feet per Pound	Ohms per 1,000 Feet	Ohms per Mile.	Feet per Ohm.	Ohms per Pound.
0000	460.000	11.684	211600.0	107.219	175	300	218	312	639.33	3359	758	4000	1 6	1.25	.04906	.25903	20383	.000076736
000	409.640	10.405	167805.0	85.028	145	245	181	262	507.01	2663	606	3200	2	1 75	.06186	.32664	16165	.00012039
00	364.800	9.266	133079.4	67.431	125	215	150	220	402.09	2112	492	2600	2.5	2	.07801	.41187	12820	.00019121
0	324.950	8.254	105592.5	53.504	100	190	125	185	319.04	1661	385	2036	3.2	2.50	.09831	.51909	10409	.00030772
1	289.300	7.348	83694.2	42.409	95	160	105	156	253.88	1328	308	1628	4	3.25	.12404	.65490	8062.3	.00048994
2	257.630	6.544	66373.0	33.632	70	135	88	131	200.54	1069	250	1310	5	4	.15640	.82582	6393.7	.00078045
3	229.426	5.827	52634.0	26.670	60	115	75	110	259.03	835	201	1060	6.3	5	.19723	1.0414	5070.2	.0012406
4	204.310	5.189	41742.0	21.151	50	100	63	92	126.12	662	163	862	8	6	.24869	1.3131	4021.0	.0019721
5	181.940	4.621	33102.0	16.773	45	90	53	77	100.01	528	134	706	10	7.50	.31361	1.6558	3188.7	.0031361
6	162.020	4.115	26250.5	13.301	35	80	45	65	79.32	417	110	577	12.6	9	.39546	2.0881	2528.7	.0049908
7	144.290	3.665	20816.0	10.548	30	67	..	..	62.90	330	90	475	16	11	.49871	2.6331	2005.2	.0079294
8	128.490	3.264	16509.0	8.366	25	60	33	46	49.80	262	74	392	20	13.50	.62981	3.3261	1590.3	.012608
9	144.430	2.907	13094.0	6.635	..	50	..	..	39.56	208	62	328	25	16	.79281	4.1870	1261.3	.020042
10	101.890	2.588	10381.0	5.260	20	40	25	32	31.37	165	52	275	32	19	1.	5.2800	1000.0	.031380
11	90.742	2.305	8234.0	4.172	.	35	..	.	24.88	131	44	231	40	22.75	1.2607	6.6568	793.18	.050632
12	80.808	2.053	6529.9	3.309	15	30	17	23	19.73	103	36	193	51	27.25	1.5898	8.3940	629.02	.080583
13	71.961	1.828	5178.4	2.618	..	25	..	..	15.65	82	31	162	64	32.50	2.0047	10.585	498.83	.12841
14	64.084	1.628	4106.8	2.081	10	22	12	16	12.41	65	26	135	81	39	2.5408	13.680	395.97	.20289
15	57.068	1.450	3256.7	1.650	..	18	..	..	9.84	51.5	21	111	102	47 50	3 1150	16.477	321.02	.31658
16	50.820	1.291	2582.9	1.309	5	15	..	..	7.81	41	18	96	129	55	4.0191	21.221	248.81	.51501

TABLE No. XII.

ELECTRIC WIRING TABLE.

Pressure in Inches of Water.	Velocity in Feet per Minute.
1/32	696.78
1/16	987.66
1/8	1,393.75
3/16	1,707.00
1/4	1,971.30
5/16	2,204.16
3/8	2,414.70
1/2	2,788.74
5/8	3,118.38
3/4	3,416.64
7/8	3,690.62
1	3,946.17
1 1/4	4,362.62
1 1/2	4,836.06
1 3/4	5,224.98
2	5,587.58

TABLE No. XIII.

VELOCITY OF AIR IN FEET PER MINUTE FOR DIFFERENT PRESSURES.

Standard Rule for Figuring Steam-heating Systems.

Q. 1. By what standard rule can a low-pressure, direct steam-heating system be figured? Such as to the amount of square feet of radiation? Size of steam mains and users? Referring particularly to a one-pipe gravity-return system, such as is being put in most of the combined office and manufacturing buildings of to-day?

Q. 2. Has there ever been any special rule established as a standard by the American Society of Mechanical Engineers?

Q. 3. If any, please state what rule is generally held as a standard.

Answers.

A. 1. In designing a system of steam heating, there is such a variety of conditions to be met that it is difficult to reduce the necessary calculations to simple rules or formulas which will give reliable results in all cases.

For that reason the American Society of Heating and Ventilating Engineers has never adopted any rule or formula as standard for such calculations. However, there are certain well-established rules and formulas which give results that agree closely with standard American practice, and which are widely used by heating and ventilating engineers.

When a system of steam heating is to be designed for a given building the first, and perhaps the most important, calculation is the determination of the amount of heat which the building will lose per hour. Heat is lost by radiation from exposed wall and glass surfaces, and by the escape of warm air from the building through doors, windows and ventilating ducts. When this heat loss has been determined, a heating system must be designed to supply that amount of heat.

Professor Carpenter's Rule.

A. 2. Carpenter gives the following formula for calculating this heat loss, which gives variable results:

$H = (G + .25\ W + .02\ n\ C)\ tx$, where H = the total heat loss from the building in B. t. u. per hour; G = the number of square feet of exposed glass surface; W = the number of square feet of exposed wall surface; n = the number of times per hour that the air will be changed by leakage at doors and windows and by ventilation; C = the volume of air in the building in cubic feet; tx = desired difference in temperature between building and outside air in F.°, usually taken at 70°.

This formula gives reliable results for a room or building having a south exposure. For an east exposure add 10 per cent to the area of exposed wall and glass; for a west exposure add 20 per cent, and for a north exposure add 30 per cent. The formula is based on a wall thickness of 16 inches, but experience shows that a wall somewhat thicker or thinner than 16 inches will differ but slightly in the rate of heat transmission.

Values of n may be taken as follows: Stores, first floor, n = 2 to 3; second floor, n = 1½ to 2. Offices, first floor, n = 2 to 2½; second floor and above, n = 1½ to 2.

Public assembly rooms, n = ¾ to 2.

Large rooms with small exposure, n = ½ to 1.

Having calculated the total heat loss per hour from the building, the next step is to determine the number of square feet of radiation required to supply this amount of heat.

In a low-pressure system of steam heating (2 to 5 pounds gauge) the temperature of the steam in the radiator is assumed as 220° F. An ordinary cast-iron radiator will give off heat at the rate of 1.7 B. t. u. per

degree difference in temperature of the surrounding air and the average temperature of the steam per hour.

Therefore, such a radiator will give off approximately 1.7 (220—70) = 255 B. t. u. per hour.

Number of square feet of radiation required is therefore $R = \frac{H}{255}$

Mills' Rule.

Mills gives the following empirical rule for calculating the square feet of radiation, which may be used as an approximate check on the above formula:

"Allow 1 square foot of steam radiation for every 200 cubic feet of volume; 1 square foot for every 20 square feet of exposed wall, and 1 square foot for every 2 square feet of exposed glass." This rule usually gives low values of radiation, and should not be considered as giving more than a rough approximation.

Size of Pipes for Mains and Risers.

A. 3. The determination of the sizes of pipes for circulating mains and risers can not be reduced with any accuracy to a rational formula, for the reason that the friction in various arrangements and conditions of piping can not be determined in advance, even approximately. Experience is the only guide in selecting the proper size of pipe, and fortunately a large amount of data is available. Kent's "Mechanical Engineer's Pocket Book" gives a table showing the number of square feet of direct steam radiation which pipes of various sizes will supply. In a single-pipe, gravity-return system a pipe should be chosen from one to two sizes larger than is given in the table, which is for a two-pipe system.

Drop in a Three-phase Circuit.

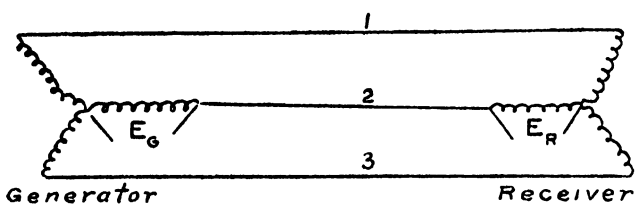
Neglecting the effect of capacity or inductance, to make the problem as simple as possible, it is desired to figure the drop in volts between a generator and its load a bank of lamps to which it supplies current.

In a single-phase circuit it is the current carried times the resistance between A and B times 2, to take care of the return wire, even as in direct currents. Now, what is the drop under similar conditions in a three-phase circuit, that is, how is the path of the return current taken into consideration?

Answer.

An alternating-current transmission circuit is usually Y-connected, because the pressure is higher and the current lower for a given amount of power than in a delta-connected system.

The figure represents a Y-connected generator supplying power to a Y-connected receiver — a trans-



former, for example. Let E_G represent the electromotive force of each winding on the generator, while E_R represents the electromotive force between one terminal and the neutral of the receiving circuit. Let the current in each line be C (a balanced load is assumed), and the resistance of each line be R.

Neglecting the effect of capacity and inductance (which must be considered in a practical solution of the problem), the drop in each line is C R. Therefore, $E_G - C\ R = E_R$ The electromotive force between lines 1 and 2, or 1 and 3, or 2 and 3, at the receiving end of the circuit is therefore $\sqrt{3}\ E_R$, where E_R is the generator voltage per phase minus the line drop in one main. The electromotive force between mains at the generator is $\sqrt{3}\ E_G$.

Therefore, to find the electromotive force per phase at the receiving end of the circuit, subtract the line drop in one main from the generator electromotive force per phase. The physical basis for this reasoning is found in the fact that the return circuit for any given main is formed by the other two mains.

COST OF POWER.

Probably the most essential thing for an operating engineer to know, especially at the present time, is how to accurately calculate the actual cost of producing his power, especially as applied to the electrical end of his plant.

It is only necessary for the operator of an isolated plant to figure his current delivered at the switchboard, but the operator of a central station must figure, in addition to this, the cost of distribution, loss of current on the line, uncollectable bills, overhead charges, billing and metering.

While the cost of power will differ for each isolated plant, depending on surrounding conditions, a fairly accurate estimate can be made that will cover all such plants.

To find the cost of operating such plants in K. W. hours, it is necessary to know:

Size of boilers.
Amount of heating surface in building.
Number of lamps and sizes.
Number of motors and sizes.
Average operating load.
Cost of coal.

The following practical question, with answers, is given below:

In the plant where I am employed there are three return tubular boilers that do nothing but furnish heat

for the plant, steam to heat water and steam for cooking purposes in the kitchen. It was originally the intention of the owners to install an engine and dynamo to furnish light for the establishment and power for three electric elevators. There are about 1,200 16-c. p. lights, and it requires about ninety H. P. for the elevators. At the present time we are consuming between forty-two and fifty tons of coal per week. Now take notice, fifty tons per week to furnish heat to the rooms and water. Instead of installing an engine and dynamo, after a room had been built for the purpose, as was intended, the owners were prevailed upon to buy their light and power.

Now, I would like to know if it would not be cheaper to install an engine and dynamo to make the light and power, heating the building with the exhaust steam, than to furnish live steam for their present use and buy the light and power? The plant is run now by two men, but if an engine and dynamo were installed, two more men (firemen) would have to be employed. What price must be paid for current in order to effect a saving over what would be saved if an engine and dynamo were installed? It seems to me that if the power and light were made here the building could practically be heated free of cost. As it now is, everything is considered a bill of expense.

Consider, also, that the boilers and pumps that are now installed in this plant were made sufficiently large to do this work, but the electric-light company persuaded the plant owners that they could sell them power cheaper than they could make it. So the owners decided to buy the current. Just what they pay I do not know, but I think about 4½ cents per K. W. hour. Our coal costs $3.50 per ton. I might add that this establishment is a seven-story hotel. Current used here is direct.

Answer No. 1.

Cost of Current, 2.7 per K. W. Hour.

The hotel would require about 350 boiler horse-power.

At the rate of 4½ pounds of coal per horse-power, it would take 141 tons per week, running twenty-four hours per day, less 50 tons which are required for heating, making 91 tons for power, or $418.50.

Allowing $30 for two firemen, the total cost would be $448.50, not including oils, grease, etc. This would make a total cost of $1,690 a month. The cost of 23,688 K. W. hours at 4½ cents would be $1,065.96, making a saving of $624.04.

This would make the current cost 2.7 cents per K. W. hour. The current, therefore, will have to be bought for at least 2½ cents to effect any saving.

Answer No. 2.

Cost of Current Less Than 2 Cents per K. W. Hour.

A plant with a fair equipment can generate current at an average cost of 2 cents or less per K. W. and heat the building.

It costs the central station 1 cent per K. W. to generate current at its own switchboard. This does not include the cost of distribution. Twelve hundred incandescent lamps will equal 72 K. W., and 90 horse-power used in the elevators will equal about 68 K. W., or 140 K. W. maximum load. It is presumed that this is seldom reached; and, if so, for only a short time each day. Still, if the plant is to be an independent one, it will have to be large enough to take care of a maximum load. It is taken for granted that the boiler-feed pumps and heater are already installed. Two 75 K. W. and one 50 K. W. machines can be installed, together with the necessary piping and wiring for

about $10,000. A good high-speed engine will develop a horse-power on about thirty-five pounds of water per hour, allowing about seven pounds of water evaporated per pound of coal, or five pounds of coal per horse-power.

During certain months of the year the exhaust may not be sufficient to heat the building; and there will be certain months before and after this period that all the exhaust may not be used for heating. It may be figured that during six months in the winter the fixed charges against the additional investment will be extra labor required, oil, waste, interest and depreciation.

For the remaining six months the fixed charges will be labor, interest and depreciation, oil and waste, and fuel over and above that used for cooking and hot water.

As no meter readings are given, and the average load also is not given, we will assume that the average load for the winter months is 80 K. W. per hour for twenty-four hours for 182 days, or a total of 349,540 K. W.

Allowing $800 for wages, $500 for interest and depreciation on investment, and $100 for oil and waste, this will be practically all it will cost to generate 349,440 K. W. during heating season. If the average load during the summer is 70 K. W. for twenty-four hours for 183 days, it would equal 307,440 K. W. Now, 1 horse-power developed on 5 pounds of coal equal 1 K. W. developed on 6¾ pounds $= \frac{6.75 \times \$3.50}{2{,}000} =$ 1.18 cents per K. W. fuel costs.

Allowing $800 for wages, $500 for interest and depreciation, and $100 for oil and waste, we have $800 + $500 + $100 = $1,400.

$$307{,}440 \times 1.18 = \$3{,}627.79 : \frac{3{,}627.79 + 1{,}400}{307{,}440} =$$

1.63 cents per K. W. during summer months.

The K. W. generated during heating season = 349,540 at a total cost of $1,400, or nearly .05 cents per K. W., or a yearly average of .84 cents per K. W., against 4½ cents charged by the central station. This low cost is reached by the reason that the boilers and pumps are necessary to furnish heat and steam, whether the power is generated on the premises or bought from the outside, and can not be charged against the electrical-generating equipment. It is a very easy matter to figure out the saving that would be made in the year.

Answer No. 3.

Cost of Current, 3 Cents per K. W. Hour.

Elevators.

Six hours' steady run taken as one day's work.

90 H. P. for 1 hour.....67,140 watt hours.
Power used in 1 day.....402,840 watt hours.
Cost for 1 day, at $0.04½ per K. W. hour.....$18.10
Power used in 1 year.....134,892,800 watt hours.
Cost for 1 year, at $0.04½ per K. W. hour...$6,610.15
H. P. hours used in 1 year.....194,400
H. P. hours reduced to B. t. u. hours.....6,474,492,000
Washington lignite contains, per lb.....11,551 B. t. u.
70 per cent loss from radiation, boilers, engine and friction, 1 lb. coal.....8,085 B. t. u.
Power developed by engine, 1 lb. coal....3,466 B. t. u.
B. t. u. hour (60 lbs. coal) contains.....207,960 B. t. u.
Tons of coal used in 12 months.....934
Cost, at $3.50 per ton.....$3,109.00

Cost, at $0.04½ per K. W. hour.....$6,610.15
Cost of power from plant..... 3,109.00

Amount saved $3,501.15

Winter Months — October 1 to April 1.

4-7 a.m. 300 lights = 900 hours, at 55 watts per light..... 49,500 watt hours.
7-8 a.m. 1,200 lights = 1,200 hours, at 55 watts per light..... 66,000 watt hours.
4-6 p.m. 600 lights = 1,200 hours, at 55 watts per light..... 66,000 watt hours.
6-9 p.m. 1,200 lights = 3,600 hours, at 55 watts per light.....198,000 watt hours.
9-12 p.m. 800 lights = 2,400 hours, at 55 watts per light.....132,000 watt hours.
12-4 a.m. 200 lights = 800 hours, at 55 watts per light..... 44,000 watt hours.

Watt hours for 1 day.....555,500 watt hours.

Cost for one day, at $0.04½ per K. W. hour.....$24.97
Watt hours for 6 months.....99,960,000 watt hours.
Cost for 6 months, at $0.04½ per K. W. hour..$4,494.00
H. P. hours for 6 months.....133,994 H. P. H.
H. P. hours reduced to B. t. u. hours.....4,562,670,170 B. t. u.
Washington lignite contains, per lb.....11,551 B. t. u.
70 per cent loss in radiation, boilers, engine and friction .....8,085 B. t. u.
Power developed by engine, 1 lb. coal.....3,466 B. t. u.
B. t. u. hour (60 lbs. coal).....207,960 B. t. u.
Tons of coal used in 6 months.....651
Cost of coal, at $3.50 per ton.....$2,278.50
Cost of two firemen, at $75 each..... 900.00
Oil, packing, repairs, etc..... 200.00

Cost of power, at $0.04½ per K. W. hour..$4,494.00
Cost of power from plant..... 3,378.50

Money saved $1,115.50

Summer Months — April 1 to October 1.

4-7 a.m. 300 lights = 900 hours, at
55 watts per light..... 49,500 watt hours.
6-9 p.m. 1,200 lights = 3,600 hours,
at 55 watts per light.....198,000 watt hours.
9-12 p.m. 800 lights = 2,400 hours,
at 55 watts per light.....132,000 watt hours.
12-4 a.m. 200 lights = 800 hours,
at 55 watts per light..... 44,000 watt hours.

Watt hours for 1 day.....423,500 watt hours.

Cost for 1 day, at $0.04½ per K. W. hour.....$18.86
Watt hours for 6 months.....76,170,000
Cost for 6 months.....$3,520.80
H. P. hours for 6 months.....102,105
H. P. hours reduced to B. t. u. hours.....3,400,607.025
Washington lignite contains, per 1 lb....11,551 B. t. u.
70 per cent loss in radiation, boilers, engine
and friction .....8,085 B. t. u.
Power developed from engine.....3,466 B. t. u.
B. t. u. hour (60 lbs. coal).....207,960 B. t. u.
Tons of coal for 6 months.....507
Cost of coal, at $3.50 per ton.....$1,774.50
Two firemen, at $75, for 6 months..... 900.00
Oil, repairs, etc..... 200.00

Cost of operating, 6 months.....$2,874.50

Cost of power, at $0.04½ per K. W. hour..$3,520.80
Cost of operating, 6 months..... 2,874.50

Money saved ..... **$,646.30**

Cost of power from plant per K. W. hour....$0.03007

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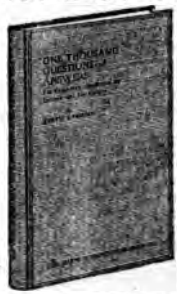
One Thousand Questions and Answers for Engineers, Applicants for License and Electricians



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